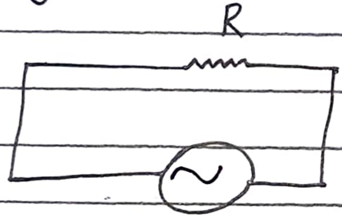
Sinosoidal Steady State circuit
AC circuit

The closed path followed by the alternating current ~~is called AC circuit or the circuit in which~~ current and voltage vary sinusoidally are called AC circuit.

All AC circuits are madeup of combination of resistor, inductor and capacitor, in each case it is assumed that purely sinusoidally alternating voltage, $V = V_m \sin\omega t$ is applied to the circuit. The equation for the current, power and phase shift are developed in each case.

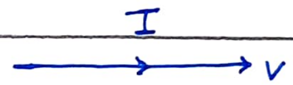
1. Purely Resistive Circuit



$V = V_m \sin\omega t$

$\therefore \quad i = \frac{V}{R}$

$i = \left(\frac{V_m}{R}\right)\sin\omega t$



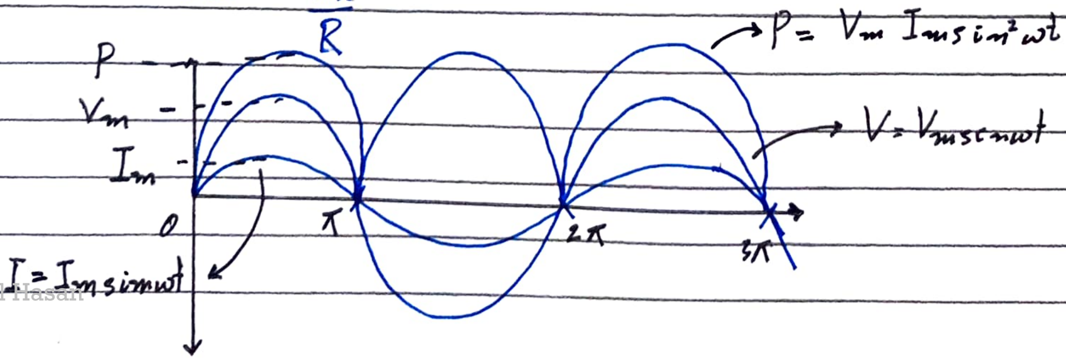
$\boxed{i = I_m \sin\omega t}$

$\phi = 0^\circ$ (same phase)

for I_m,

$\theta = 90^\circ$

$\Rightarrow \quad I_m = \frac{V_m}{R}$



Let the applied voltage:

$$V = V_m \sin\omega t \quad — (1)$$

According to Ohm's Law, we can find the equation for current, i.e

$$i = \frac{V}{R}$$

$$\text{or} \quad i = \frac{V_m \sin\omega t}{R}$$

The instantaneous current i, is maximum when $\sin\omega t = 1$ or $\theta = 90°$ since the maximum value,

$$I_M = \frac{V_M}{R}.$$

Hence the instantaneous current is written by the equation

$$i = I_M \sin\omega t \quad — (2)$$

from eq (1) and (2) the alternating voltage and current has same frequency and phase therefore we can say that the alternating current and voltage are in same phase with each other.

Now,

$$p = V \times i$$

$$p = V_m \sin\omega t \times I_m \sin\omega t$$

$$p = V_m I_m \sin^2\omega t$$

$$p = V_m I_m \left(\frac{1 - \cos 2\omega t}{2}\right)$$

$$p = V_m I_m - \underbrace{\frac{V_m I_m \cos 2\omega t}{2}}_{Av = 0}$$

$$p_{av} = \frac{V_m I_m}{2}$$

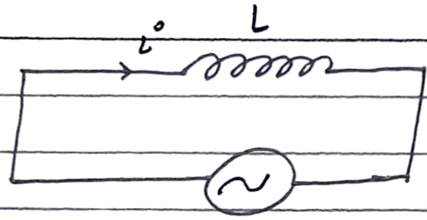
$$P_{av} = \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}} \longrightarrow \text{rms values}$$

$$\therefore \boxed{P_{av} = VI}$$

The power consist of constant part $\frac{V_m \times I_m}{2}$ and fluctuating part with double frequency that of voltage and current. The average value of fluctuating part is always zero. Hence the average value of power.

$$P_{av} = \frac{V_m \times I_m}{2}$$

2. Purely Inductive Circuit



$V = V_m \sin\omega t$

$$e_L = -L\frac{di}{dt}$$

Applying KVL

$$\therefore \quad V + e_L = 0$$

$$V = L\frac{di}{dt}$$

$$V_m \sin\omega t = L\frac{di}{dt}$$

$$\int di = \frac{V_m}{L}\int \sin\omega t \, dt$$

$$i = \frac{V_m}{\omega L}(-\cos\omega t)$$

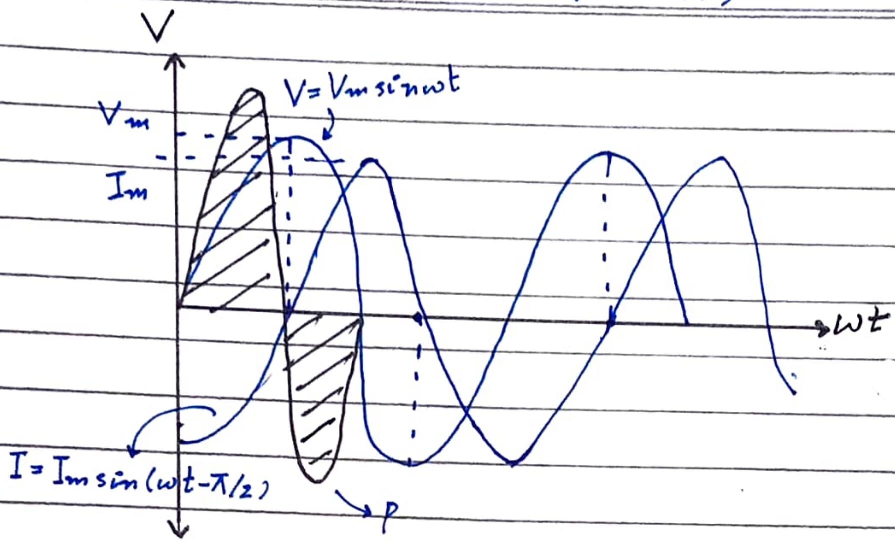
$$\boxed{i = \frac{V_m}{\omega L}\sin(\omega t - \pi/2)}$$

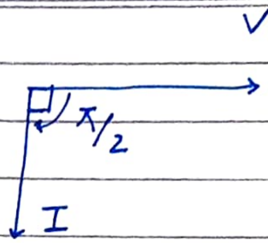
$X_L = \omega L$ Inductive reactance

$$\Rightarrow \quad i = \frac{V_m}{X_L}\sin(\omega t - \pi/2)$$

When, $\sin(\omega t - \pi/2) = 1$, $I_m = \dfrac{V_m}{X_L}$

$$I = I_m \sin(\omega t - \pi/2)$$



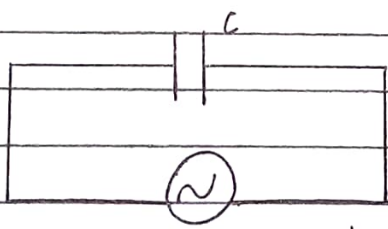


$$p = Vi$$
$$= V_m \sin\omega t \times I_m(\omega t - \pi/2)$$
$$= \frac{V_m I_m}{2} \sin 2\omega t$$

$$p_{av} = 0.$$

When power term is 'positive', energy gets stored in magnetic field due to the increasing current but due to the negative cycle in the power curve, this power is returned to supply.
Hence, the average power observed in pure inductive circuit is always 0.

Purely Capacitive Circuit



$$V = V_m \sin\omega t \quad \text{--- (1)}$$

$i = \dfrac{dq}{dt}$, and here

$$Q = CV$$

$$\Rightarrow i = \frac{d(CV)}{dt}$$

$$i = \frac{d(CV_m \sin \omega t)}{dt}$$

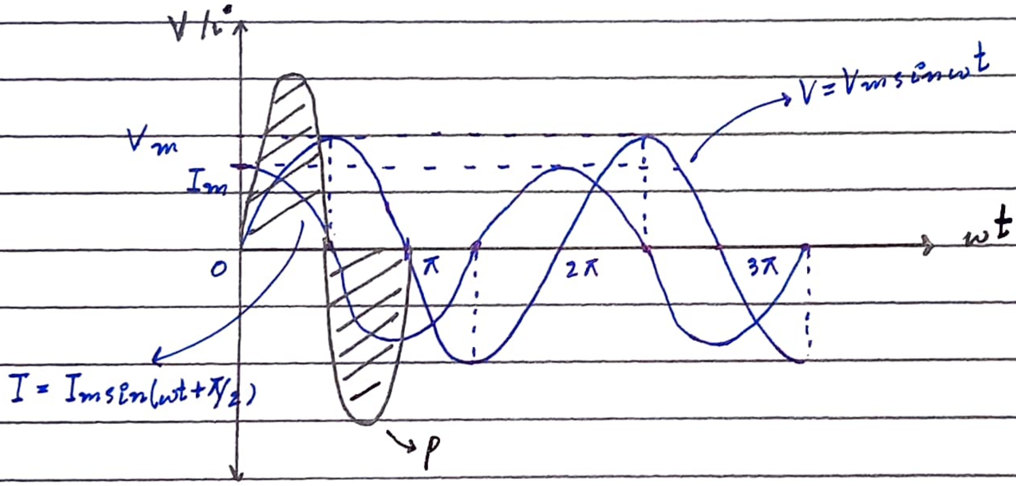
$$i = CV\omega \cos \omega t$$

$$i = \frac{V}{1/\omega C} \sin(\omega t + \pi/2)$$

$$\Rightarrow \quad i = \frac{V}{X_C} \sin(\omega t + \pi/2) \quad \text{—— (2)}$$

From eq (1) and (2), the alternating voltage and current have different phase i.e phase difference of $\pi/2$ rad or $90°$.

Therefore, the current is said to be leading ahead the voltage.



$$\therefore \quad P = VI$$

$$= V_m \sin\omega t \, I_m \sin(\omega t + \pi/2)$$

$$P_{avg} = \frac{V_m I_m}{2} \sin 2\omega t$$

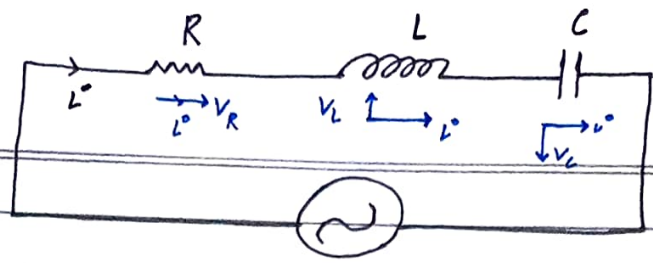
$$P_{avg} = 0$$

Hence, the power equation is purely sinusoidal of double frequency at an applied voltage.

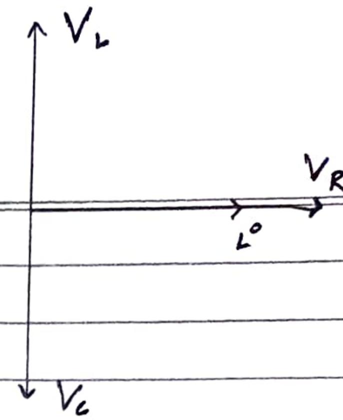
Therefore, the average value of power over 1 complete cycle is 0.

When power curve is positive the energy gets stored in the capacitor during it's charging condition, while in the negative power curve, the stored energy is returned back to supply, during it's discharging.

Series RLC circuit



$V = V_m \sin\omega t$



Series RLC circuit are extensively used in electrical circuit and hence their analysis is very important and it maybe noticed that all the AC quantities are vector quantities, they have both magnitude and direction. Either polar or rectangular these forms are extensively used in the analysis of AC circuits.

A series having resistance R, inductance L and capacitance C and applied voltage $V = V_m \sin\omega t$. Therefore, series RLC circuit on a current i. Due to this current, there are different voltage across the resistor, inductor and capacitor.

Now,

the voltage drop across resistance 'R';

$V_R = \overrightarrow{I \cdot R}$ (in phase with it)

the voltage drop across inductor 'L';

$V_L = I \cdot X_L$ (leading by $\pi/2$)

and the voltag drop across capacitor 'C';

$V_C = I \cdot X_C$ (lagging with current by an angle of $\pi/2$)

There are 4 voltages in series RLC circuit i.e V_R, V_L, V_C and resultant voltage V.

According to KVL, the resultant voltage

$V = V_R + V_L + V_C$

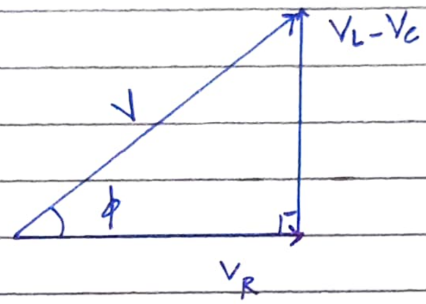
But, V_L and V_C are in opposite direction.

The resultant of V_L and V_C is the arithmetic difference between them.

Therefore, three possible cases arises in series RLC circuit.

(i) $V_L > V_C$
(ii) $V_C > V_L$
(iii) $V_L = V_C$

Case 1: $V_L > V_C$



$\therefore$ Using Pythagores Theorem

$\Rightarrow V^2 = V_R^2 + (V_L - V_C)^2$

$\Rightarrow V = \sqrt{V_R^2 + (V_L - V_C)^2}$

$$\therefore \quad V = \sqrt{(IR)^2 + (IX_L - IX_C)^2}$$

$$V = I\sqrt{R^2 + (X_L - X_C)^2}$$

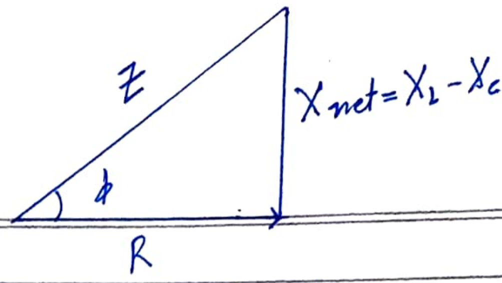
$$\Rightarrow \quad V = IZ.$$

where, $Z = \sqrt{R^2 + (X_L - X_C)^2}$
is known as the circuit impedence of series RLC circuit and
$X_{net} = X_L - X_C$ is called net reactance of series RLC circuit.

Impedence is defined as the total opposition offered to the current flow due to resistance, inductive reactance and capacitance reactance of the series RLC circuit. It is expressed in Ω.

$$\tan\phi = \frac{X_{net}}{R} = \frac{X_L - X_c}{R}$$

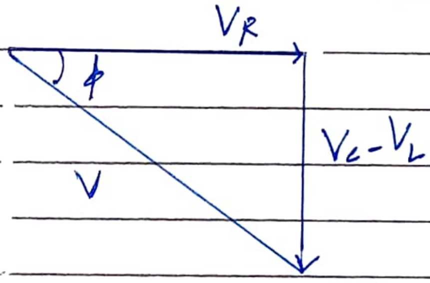
$$\phi = \tan^{-1}\left(\frac{X_L - X_c}{R}\right) \rightarrow \text{positive}$$



If $\omega L > \frac{1}{\omega C}$ and then the phase angle is positive, an then series RLC circuit becomes purely inductive circuit.

So, if ; $V = V_m \sin\omega t$, then the instantaneous current $I = I_m \sin(\omega t - \phi)$ because current lags behind the applied voltage by an angle ϕ.

(ii) $V_c > V_L$



Using Pythagores Theorem ;

$$V^2 = V_R^2 + (V_c - V_L)^2$$

$$V = \sqrt{V_R^2 + (V_c - V_L)^2}$$

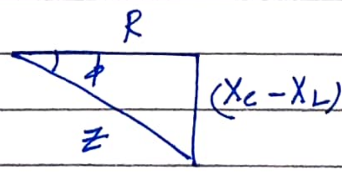
$$V = \sqrt{(IR)^2 + (IX_c - IX_L)^2}$$

$$V = I\sqrt{(R)^2 + (X_c - X_L)^2}$$

$$V = I \cdot Z$$

here

$$Z = \sqrt{R^2 + (X_c - X_L)^2} \longrightarrow \text{Impedence}$$



$$\tan\phi = \frac{X_{net}}{R} = \frac{X_c - X_L}{R}$$

$$\phi = \tan^{-1}\left(\frac{X_c - X_L}{R}\right) \rightarrow \text{negative}$$

$$\Rightarrow \frac{1}{\omega C} > \omega L$$

(مسلسل) $V_L = V_C$

This condition is the resonance.

A series RLC circuit is said to be in electrical resonance when the inductive ~~reasona~~ reactance is equal to the capacitive reactance X_C and it's net reactance is 0.

Resonance is defined as the condition in a circuit containing atleast one inductor and one capacitor and when the supply voltage and supply current are in same phase.

$$\therefore \quad Z = \sqrt{R^2 + (X_L - X_C)^2}$$

As $X_L = X_C$

$$\Rightarrow \quad Z = R$$

$\therefore$ The total impedence of series RLC circuit at resonance condition $Z = R$.

It is also known as circuit impedence at resonance is called dynamic impedence.

$$X_L = X_C$$

$$\omega L = \frac{1}{\omega C}$$

$$\Rightarrow \quad \omega^2 = \frac{1}{LC}$$

$$\omega_R = \sqrt{\frac{1}{LC}}$$

$$f_R = \frac{1}{2\pi\sqrt{LC}}$$

Resonance frequency

The frequency at which resonance occurs, is called resonance frequency. The series resonance effect maybe produced by the following methods.

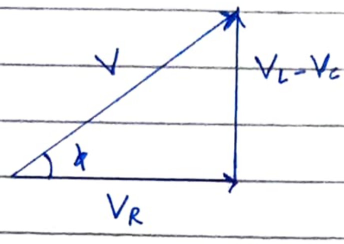
1. Varying frequency, keeping inductance and capacitance constant.

2. Varying either L or C for a given frequency.

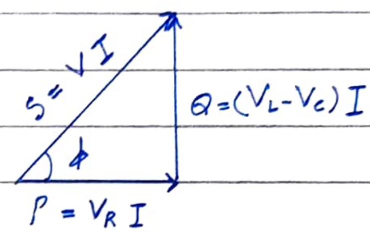
Circuit current is maximum => $I_M = \dfrac{V_m}{R}$

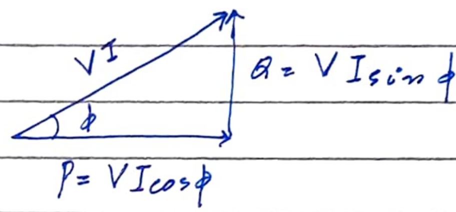
Power in series RLC circuit.

$V_L > V_C$



=>





For any condition $X_L > X_C$ or $X_L < X_C$ in general power can be expressed as:

P = voltage × component of current in phase with voltage

∴ Power consumed by series RLC circuit is:

$$P = VI\cos\phi$$

The power drawn by AC circuit can be of 3 types:

(i) Operand Power (S): It is the product of RMS value of applied voltage and circuit current.

$$P = VI \text{ (volt-ampere)}$$

(ii) Active Power (P): It is the power which is actually dissipated in the circuit resistance.

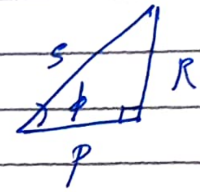
$$P = VI\cos\phi \text{ (watt).}$$

(iii) Reactive Power (Q): It is the power dissipated in inductive or capacitance reactance of the series RLC circuit and also defined as product of applied voltage and reactive component of current.

$$P = VI\sin\phi \text{ (volt-ampere)}$$

From power Δ, these powers are related as

$$S^2 = P^2 + Q^2$$



→ Power Factor:

The power factor of series RLC circuit is a measure of it's effectiveness in utilizing the operand power drawn by the AC circuit or the ratio of active power to the operand power in series RLC circuit is defined as the power factor

$$\text{Power factor } (P_f) = \frac{\text{Active power}}{\text{Apparent Power}} = \frac{VI\cos\phi}{VI} = \cos\phi$$

$$\boxed{P_f = \cos\phi = R/Z}$$

At resonance;

$$\cos\phi = 1$$

$$\boxed{\phi = 0^\circ}$$

→ Quality factor

The quality factor of series RLC circuit indicate how many times the potential difference across inductor or capacitor is greater than the applied voltage.

$$\text{Quality Factor } (Q) = \frac{\text{Voltage across L or C}}{\text{Applied voltage}}$$

$$\therefore \quad Q = \frac{I \cdot X_L}{I \cdot Z} = \frac{X_L}{Z} \text{ OR } \frac{I \cdot X_C}{Z} = \frac{X_C}{Z}$$

$$\therefore \quad Q = \frac{\omega_R L}{Z} \quad \text{or} \quad \frac{1}{\omega_R C Z}$$

$$Q = \frac{\omega_R L}{R} \quad \left(\omega_R = \sqrt{\frac{1}{LC}} \rightarrow \text{Resonance frequency}\right)$$

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

$(0 \rightarrow 100) \rightarrow$ Range (Quality factor)

$(0 \rightarrow 1) \rightarrow$ Range

(Power factor)

Q. 230 V, 50 Hz AC supply is applied to a coil of 0.06 H inductance and 2.5 Ω resistance connected in series with 6.8 μF capacitance.

Calculate

(i) Circuit Impedence

(ii) Circuit current

(iii) Phase angle between voltage and current

(iv) Total power consumed and power factor.

$\therefore$ (i) $L = 0.06\,H$, $C = 6.8 \times 10^{-6} F$

$f = 50\,Hz$

$$\Rightarrow X_L = 2\pi f L = 2 \times \pi \times 50 \times 0.06 = 18.84\,\Omega$$

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2 \times \pi \times 50 \times 6.8 \times 10^{-6}} = 468.102\,\Omega$$

$$\therefore Z = \sqrt{R^2 + (X_C - X_L)^2}$$

$$Z = \sqrt{(2.5)^2 + (449.262)^2} = 449.268\,\Omega$$

(ii) $I_M = \frac{V_M}{Z} = \frac{230}{449.268} = 0.511\,A$ ans.

(iii) $\tan\phi = \frac{X_{net}}{R} = \frac{X_C - X_L}{R}$

$= \frac{449.262}{2.5}$

$\therefore \tan\phi = 179.704$

$\phi = \tan^{-1}(179.704)$

$\phi = 89.68°$

(iv) Power factor $= \cos\phi = \cos(89.68°) = 0.0058$

$\therefore P = VI\cos\phi$

$= 230 \times 0.511 \times \cos(89.68)$

$P = 0.656\ W$ Ans.

Q. A coil having a resistance 7Ω and inductance of 31.8 mH is connected to a 230 V and 50 Hz AC supply. Calculate

(i) Circuit current

(ii) Phase angle between voltage and current

(iii) Total power consumed

(iv) Power factor

$\therefore X_L = 2\pi f L = 2 \times \pi \times 50 \times 31.8 \times 10^{-3}$

$X_L = 9.990\ \Omega$

$\therefore Z = \sqrt{R^2 \quad X_L^2}$

$Z = \sqrt{7^2 + (9.99)^2}$

$Z = 12.19\ \mathring{A}$

(i) $I = \frac{V}{Z} = \frac{230}{12.19} = 18.86\ A$

(ii) $\tan\phi = \frac{X_L}{R} = 1.42$

$\phi = \tan^{-1}(1.42) = 54.8°$ Ans

(iii) and (iv)

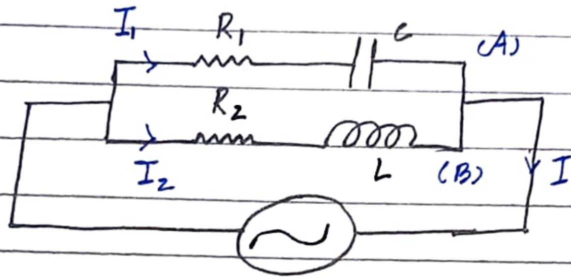
Power factor $\rightarrow$ $\cos\phi = \cos(54.8) = 0.576$

$\therefore P = VI\cos\phi$

$= 230 \times 18.86 \times \cos(54.8)$

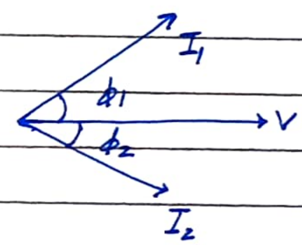
$= 2500.448$ W **Ans.**

→ Parallel RLC circuit



$V = V_m \sin\omega t$

A parallel circuit consists of two branches connecting in parallel. Voltage is same for each branch but the branch current may differ in magnitude and phase depending on the branch impedence.



1. For branch A

$Z_1 = \sqrt{R^2 + X_C^2}$

$I_1 = \frac{V}{Z_1}$

$\phi_1 = \tan^{-1}\left(\frac{X_C}{R_1}\right)$

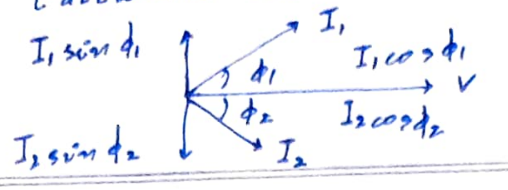
2. For branch B

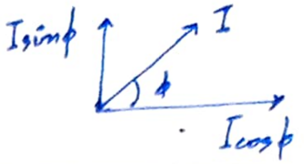
$Z_2 = \sqrt{R^2 + X_L^2}$

$I_2 = V/Z_2$

$\phi_2 = \tan^{-1}\left(\frac{X_L}{R}\right)$

Calculate the resultant current I





$$\Rightarrow \quad I\cos\phi = I_1\cos\phi_1 + I_2\cos\phi_2$$

$$I\sin\phi = I_1\sin\phi_1 - I_2\sin\phi_2$$

$$I = \sqrt{(I\cos\phi)^2 + (I\sin\phi)^2}$$

$$\phi = \tan^{-1}\left(\frac{Y\text{-comp}}{X\text{comp}}\right) = \tan^{-1}\left(\frac{I\sin\phi}{I\cos\phi}\right)$$

If $\tan\phi$ is +ve, then the resultant current I leads ahead the voltage and $\tan\phi$ is negative. Then the resultant current I lags behind the voltage.

$$\text{Power Factor} = \cos\phi = \frac{I\cos\phi}{I} = \frac{I_1\cos\phi_1 + I_2\cos\phi_2}{I}$$

$$\text{Quality Factor} = \frac{I_L}{I} \quad \text{or} \quad \frac{I_C}{I_L}$$

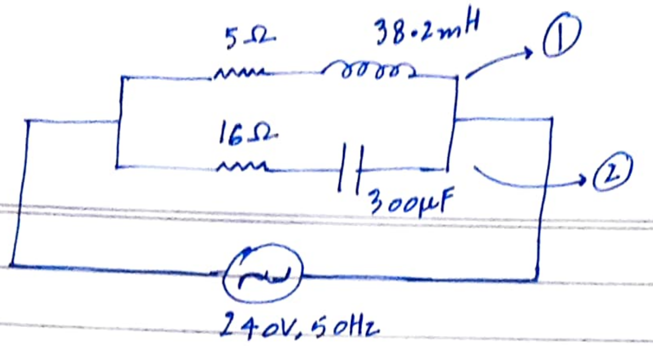
Q. A parallel circuit consists of two branches, one containing a coil a resistance 5 Ω and inductance 38.2 mH, the other non-inductive resistance 16 Ω in series with capacitor of 300 μF capacitance. The circuit connected to a 240V and 50Hz AC supply. Determine;

(i) Current in each branch

(ii) Total resultant current

(iii) Phase angle between voltage and resultant current

(iv) Power factor and Quality factor



$\therefore$ For branch ①

$X_L = 2\pi f L = 2 \times \pi \times 50 \times 38.2 \times 10^{-3}$

$X_L = 12\ \Omega$

$\therefore$ $Z_1 = \sqrt{R^2 + X_L^2} = \sqrt{5^2 + 12^2}$

$Z_1 = 13\ \Omega$

$I_1 = \dfrac{V}{Z_1} = \dfrac{240}{13} = 18.46\ A$

$\therefore$ $\tan\phi_1 = \dfrac{X_L}{R} = 2.6$

$\phi_1 = \tan^{-1}(2.6) = 68.96^\circ$

For branch ②

$X_c = \cancel{2\pi}\ \dfrac{1}{2\pi f C} = \dfrac{1}{2 \times \pi \times 50 \times 300 \times 10^{-6}}$

$X_c = 10.61\ \Omega$

$\therefore$ $Z_2 = \sqrt{R^2 + X_c^2} = \sqrt{16^2 + (10.61)^2} = 19.19\ \Omega$

$\Rightarrow$ $I_2 = \dfrac{V}{Z_2} = \dfrac{240}{Z_2} = \dfrac{240}{19.19} = 12.50A$

$\therefore$ $\tan\phi_2 = \dfrac{X_c}{R} = 0.66$

$\phi_2 = \tan^{-1}(0.66) = 33.42^\circ$ Ans

$I\cos\phi = I_1\cos\phi_1 + I_2\cos\phi_2$

$= 18.46\cos(68.96) + 12.50\cos(0.66)$

$= 6.62 + 12.49$

$I\cos\phi = 19.11$

$I\sin\phi = I_1\sin\phi_1 - I_2\sin\phi_2$

$= 18.46\sin(68.96) + 12.50\sin(0.66)$

$= 17.22 + 0.14$

$I\sin\phi = 17.36$

$\Rightarrow \quad I = \sqrt{(I\cos\phi)^2 + (I\sin\phi)^2}$

$I = \sqrt{(19.11)^2 + (17.36)^2}$

$I = 25.81\,A$

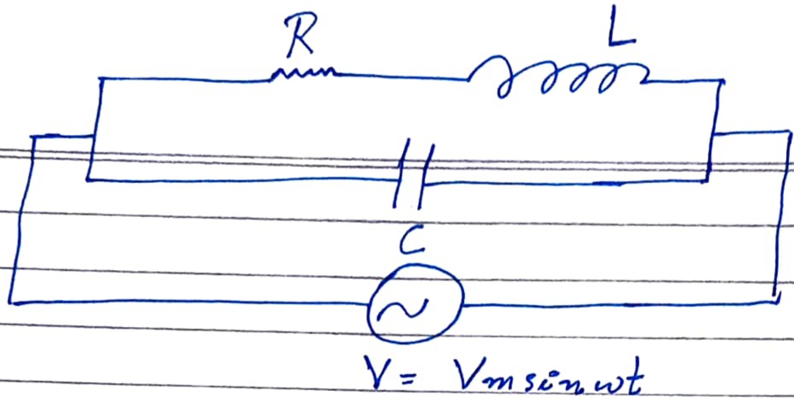
$\therefore \quad \tan\phi = \frac{I\sin\phi}{I\cos\phi} = \frac{17.36}{19.11} = 0.908$

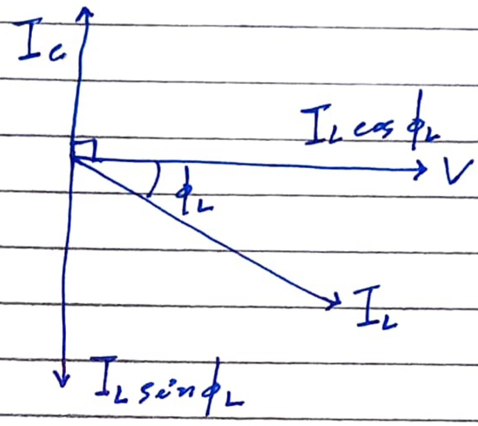
$\phi = \tan^{-1}(0.908) = 42.23^\circ$ Ans.

$\therefore$ Power factor $= \cos\phi = \frac{I\cos\phi}{I} = \frac{19.11}{25.81} = 0.74$

Quality factor $= \frac{I_L}{I} = \frac{18.46}{25.81} = 0.715$ Ans.

Resonance in Parallel RLC circuit ;





Consider a practical case of parallel RLC circuit consisting of a resistance R in series with inductor L and both connected parallel to a capacitor C across the AC voltage $V = V_m \sin\omega t$, the current flows in inductive branch I_L and capacitive branch I_c ;

$$I_L = V/Z_L$$

$$Z_L = \sqrt{R^2 + X_L^2}$$

$$\phi_L = \tan^{-1}\left(\frac{X_L}{R}\right)$$

and $$I_c = \frac{V}{X_c}$$

the current I_L lags behind the applied voltage V by an angle ϕ_L while the current I_c leads, the applied voltage V by an angle of 90°. Therefore, such a circuit is in resonance when the reactive component of line current is 0 i.e

$$I_c - I_L \sin\phi = 0$$
$$I_c = I_L \sin\phi$$

$$\frac{V}{X_c} = \frac{V}{X_L} \sin\phi$$

$$\omega C = \frac{1}{\sqrt{R^2 + X_L^2}} \sin\phi$$

$$\omega C = \frac{1}{\sqrt{R^2+X_L^2}} \times \frac{X_L}{Z_L}$$

$$Z_L^2 = X_L \cdot X_c$$
$$R^2 + \omega^2 L^2 = X_L \cdot X_c$$
$$R^2 + X_L^2 = \frac{L}{C}$$
$$X_L^2 = \frac{L}{C} - R^2$$

$$\boxed{\omega = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L}}}$$

$$\omega = \frac{1}{2\pi}\sqrt{\frac{1}{LC}} \;; \text{ if } \frac{R^2}{L^2} << \frac{1}{LC}$$

then,

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}$$

If resistance R of the parallel RLC circuit is very small or $\frac{R^2}{L^2} <<< \frac{1}{LC}$. then the resonance frequency of parallel RLC circuit $f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}$ which is same as resonance frequency of series RLC circuit.

Quality factor : $\dfrac{I_L}{I_L \cos\phi_L} = \dfrac{1}{\cos\phi_L} = \sec\phi_L$

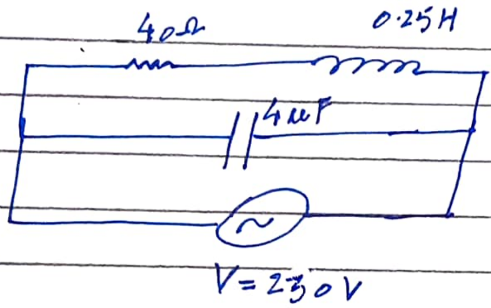
Q. A circuit consists of $4\mu F$ capacitor in parallel with coil of resistance 40Ω and inductance $0.25H$. If a voltage applied to the circuit and parallel AC circuit is $230V$ at resonance frequency f_0. Calculate

(i) current in each branch

(ii) resultant current

(iii) phase angle between voltage and current

(iv) Quality factor and power factor.



$$X_L = 2\pi f L$$
$$= 2 \times \pi \times 157.1 \times 0.25$$
$$= 246.77\,\Omega$$

First calculate f_0 then put f_0 in X_L

and $f_0 = \dfrac{1}{2\pi}\sqrt{\dfrac{1}{LC} - \dfrac{R^2}{L^2}}$

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{0.25 \times 4 \times 10^{-6}} - \frac{40^2}{(0.025)^2}} = 157.104\ Hz$$

$$I_L = \frac{V}{Z_L} = \frac{230}{\sqrt{40^2 + 246.77^2}} = 0.960\,A$$

$$I_c = \frac{V}{X_c} = 230 \times 2 \times \pi \times 157.104 \times 4 \times 10^{-6} = 0.908\,A$$

(ii) $I = I_L \cos\phi_L = 0.960 \cos\left(\dfrac{246.77}{40}\right) = 0.147\ A$ Ans

(iii) At resonance :

$\phi = 0°$

Power factor = $\cos\phi = 1$

(iv) Quality factor = $\sec\phi_L = \sec(80.78) = 6.241$

Ans

These handwritten notes are of ESC-S101 taught to us by Prof. Om Pal, compiled and organized chapter-wise to help our juniors. We hope they make your prep a bit easier.

— **Saksham Nigam** and **Misbahul Hasan** (B.Tech. CSE(2024-28))