Network Theory

1. Circuit Theory Analysis – Mesh Analysis and Nodal Analysis.
2. Star- Delta Transformation
3. Network Theorem - Superposition Theorem, Norton Theorem and Maximum Power transfer theorem.

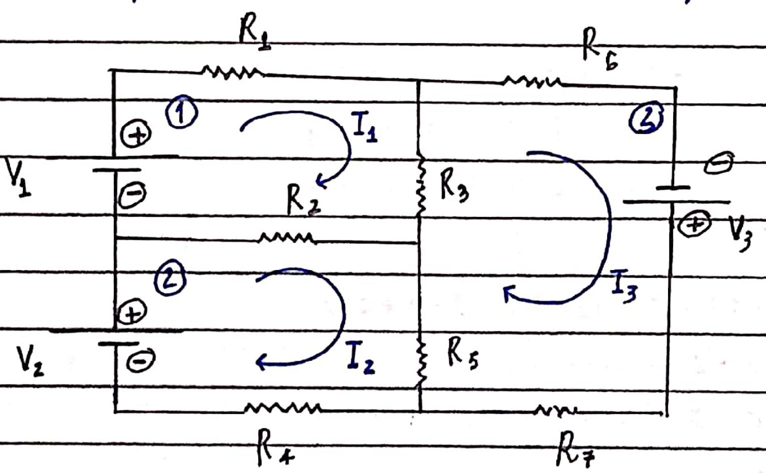
→ Kirchoff's Voltage Law: The algebraic sum of voltage drop in any closed path in a specified direction is equal to 0.

→ Kirchoff's Current Law: The algebraic sum of incoming and outgoing current in a junction is 0.

1. Mesh Analysis:

Mesh or loop analysis is based on Kirchoff's voltage law, it is used to find out unknown current, voltage in the mesh.

In this method, each mesh is assigned a seperate mesh current and KVL is applied to write the mesh equation.



$R_6 I_3 + R_7 I_3 + R_5 (I_3 - I_2) + R_3 (I_3 - I_1) = 0$

$-I_1 R_3 - I_2 R_5 + I_3 (R_3 + R_5 + R_6 + R_7) = V_3$ — (3)

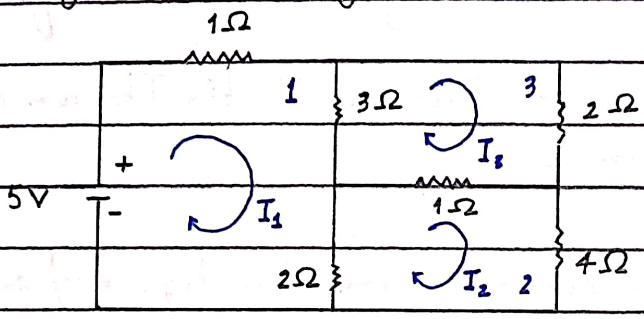
$I_1 R_1 + R_3 (I_1 - I_3) + R_2 (I_1 - I_2) = V_1$

$I_1 (R_1 + R_2 + R_3) - I_2 R_2 - I_3 R_3 = V_1$ — (1)

$R_2 (I_2 - I_1) + R_5 (I_2 - I_3) + R_4 I_2 = V_2$

$-R_2 I_1 + I_2 (R_2 + R_5 + R_4) - I_3 R_5 = V_2$ — (2)

Q. Find the current I_1, I_2 and I_3 of the given network using mesh analysis.



$$5I_1 + 3(I_1 - I_3) + 2(I_1 - I_2) = 5$$
$$I_1(10) - I_2 \cdot 2 + 5I_1 = 5$$
$$10I_1 - 2I_2 - 3I_3 = 5 \quad \text{--- (1)}$$

$$3(I_3 - I_1) + 1(I_3 - I_2) + 2I_3 = 0$$
$$6I_3 - 3I_1 - I_2 = 0 \quad \text{--- (3)}$$

$$1(I_2 - I_3) + 4I_2 + 2(I_2 - I_1) = 0$$
$$-2I_1 + 7I_2 - I_3 = 0 \quad \text{--- (2)}$$

Arranging the mesh equations into matrix form and using Crammer's rule.

$$[I][R] = [V]$$

$$\Rightarrow \begin{bmatrix} 6 & -2 & -3 \\ -2 & 7 & -1 \\ -3 & -1 & 6 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ 0 \end{bmatrix}$$

$$\therefore \Delta = 6(42-1) + 2(-12-3) - 3(2+21)$$
$$= 6(41) + 2(-15) - 3(23)$$
$$= 246 - 30 - 69$$
$$= 147$$

$$\therefore \begin{bmatrix} 5 & -2 & -3 \\ 0 & 7 & -1 \\ 0 & -1 & 6 \end{bmatrix}, \quad \Delta_1 = 5(41) + 2(0) - 3(0)$$
$$= 205$$

$$\Delta_2 = \begin{vmatrix} 6 & 5 & -3 \\ -2 & 0 & -1 \\ -3 & 0 & 6 \end{vmatrix} = 6(0) - 5(-15) - 3(0) = +75$$

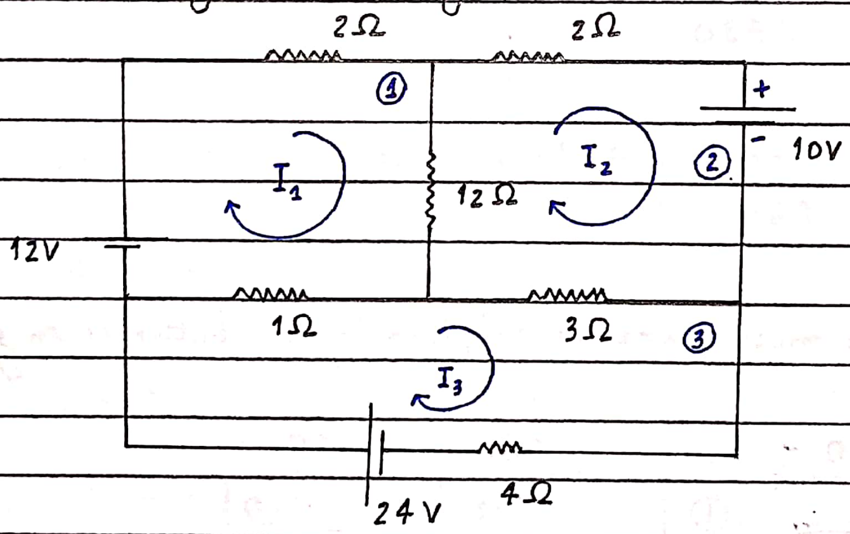
$$\Delta_3 = \begin{vmatrix} 6 & -2 & 5 \\ -2 & 7 & 0 \\ -3 & -1 & 0 \end{vmatrix} = 6(0) + 2(0) - 5(23) = -115$$

$$\Rightarrow \quad I_1 = \frac{\Delta_1}{\Delta} = \frac{205}{147} = 1.39\ A$$

$$I_2 = \frac{\Delta_2}{\Delta} = \frac{75}{147} = 0.51\ A$$

$$I_3 = \frac{\Delta_3}{\Delta} = \frac{-115}{147} = -0.78\ A \quad \text{ans}$$

Q. Determine the branch current in 4Ω resistor in the given network using mesh analysis.



$$\therefore \quad 2I_1 + 12(I_1 - I_2) + 1(I_1 - I_3) = 12$$

$$15I_1 - 12I_2 - I_3 = 12 \quad \text{— (1)}$$

$$2I_2 + 12(I_2 - I_1) + 3(I_2 - I_3) = -10$$

$$-12I_1 + 17I_2 - 3I_3 = -10 \quad \text{— (2)}$$

$$1(I_3 - I_1) + 3(I_3 - I_2) + 4I_3 = 24$$

$$-I_1 - 3I_2 + 8I_3 = 24 \quad \text{— (3)}$$

Arranging mesh equations into matrix form and using crammer's rule.

$$\therefore \quad [R][I] = [V]$$

$$\begin{bmatrix} 15 & -12 & -1 \\ -12 & 17 & -3 \\ -1 & -8 & 8 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 12 \\ -10 \\ 24 \end{bmatrix}$$

$$\therefore \quad \Delta = \begin{vmatrix} 15 & -12 & -1 \\ -12 & 17 & -3 \\ -1 & -3 & 8 \end{vmatrix} = 15(17\cdot 8 - 9) + 12(-96-3) - 1(36+1$$

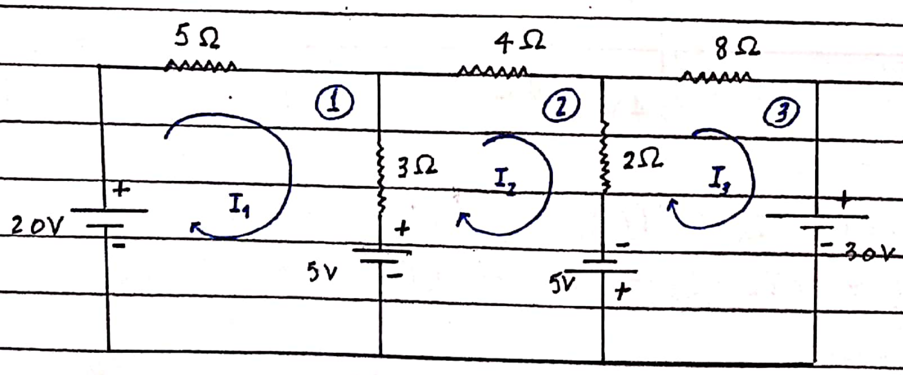
$$\therefore \quad \Delta = 664$$

$$\Delta_3 = \begin{vmatrix} 15 & -12 & 12 \\ -12 & 17 & -10 \\ -1 & -3 & 24 \end{vmatrix} = 15(17\cdot 24 - 30) + 12(-12\cdot 24 - 10) + 12(36+17)$$

$$\Delta_3 = 2730$$

$$\Rightarrow \quad I_3 = \frac{2730}{664} = 4.11 \text{ A} \quad \text{ans}$$

Q. Determine the mesh current supplied by the batteries in given network.



$$5I_1 + 3(I_1 - I_2) = 20 - 5$$
$$8I_1 - 3I_2 = 15 \quad \text{— (1)}$$

$$4I_2 + 3(I_2 - I_1) + 2(I_2 - I_3) = 5 + 5 + 5$$
$$-3I_1 + 9I_2 - 2I_3 = 15 \quad \text{— (2)}$$

$$8I_3 + 2(I_3 - I_2) = -30 - 5$$
$$-2I_2 + 20I_3 = -35 \quad \text{— (3)}$$

Arranging the mesh equations into matrix form and using Crammer's Rule.

$$\begin{bmatrix} 8 & -3 & 0 \\ -3 & 9 & -2 \\ 0 & -2 & 10 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 15 \\ 15 \\ -35 \end{bmatrix}$$

$\therefore \quad \Delta = 8(90-4)+3(-30)$

$= 8(86)-90$

$= 598$. ans

$$\Delta_1 = \begin{vmatrix} 15 & -3 & 0 \\ 15 & 9 & -2 \\ -35 & -2 & 10 \end{vmatrix} = 15(86)+3(150-70) = 1050$$

$$\Delta_2 = \begin{vmatrix} 8 & 15 & 0 \\ -3 & 15 & -2 \\ 0 & -35 & 10 \end{vmatrix} = 8(150-70)-15(-30)$$

$= 8(80) + 450$

$= 640+450 = 1090$

$$\Delta_3 = \begin{vmatrix} 8 & -35 & 15 \\ -3 & 9 & 15 \\ 0 & -2 & -35 \end{vmatrix} = 8(9(-35)-30)+35(3\times35)+15(6)$$

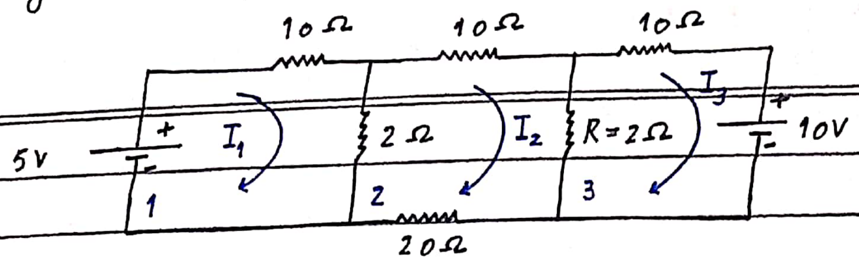
$= 1005$

$\Rightarrow \quad I_1 = \dfrac{\Delta_1}{\Delta} = \dfrac{1050}{598} = 1.755\ \dot{A}$

$I_2 = \dfrac{\Delta_2}{\Delta} = \dfrac{1090}{598} = 1.822\ \dot{A}$

$I_3 = \dfrac{\Delta_3}{\Delta} = \dfrac{1005}{598} = 1.68\ \dot{A}$ ans

Q. Find the voltage across the resistor R using Mesh Analysis.



Applying mesh analysis in mesh 1

$$-10I_1 - 2(I_1 - I_2) + 5 = 0$$
$$-12I_1 + 2I_2 = -5$$
$$12I_1 - 2I_2 = 5 \quad \text{——} \quad (1)$$

Applying mesh analysis in mesh 2

$$-10I_2 - 2(I_2 - I_3) - 20I_2 - 2(I_2 - I_1) = 0$$
$$-34I_2 + 2I_3 + 2I_1 = 0$$
$$I_1 - 17I_2 + I_3 = 0 \quad \text{——} \quad (2)$$

Applying mesh analysis in mesh 3

$$-10 - 2(I_3 - I_2) - 10I_3 = 0$$
$$+2I_2 - 12I_3 = 10$$
$$I_2 - 6I_3 = 5 \quad \text{——} \quad (3)$$

Arranging mesh equations into matrix form and applying crammer's rule.

$$\therefore \begin{bmatrix} 12 & -2 & 0 \\ 1 & -17 & 1 \\ 0 & 1 & -6 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ 5 \end{bmatrix}$$

$$\therefore \Delta = \begin{vmatrix} 12 & -2 & 0 \\ 1 & -17 & 1 \\ 0 & 1 & -6 \end{vmatrix} = 12(17(6)-1) + 2(-6)$$
$$12(101) - 12$$
$$= 1200.$$

$$\Delta_2 = \begin{vmatrix} 12 & 5 & 0 \\ 1 & 0 & 1 \\ 0 & 5 & -6 \end{vmatrix} = 12(-5) - 5(-6) = -60 + 30 = -30$$

$$\Delta_3 = \begin{vmatrix} 12 & -2 & 5 \\ 1 & -17 & 0 \\ 0 & 1 & 5 \end{vmatrix} = 12((-17)(5)) + 2(5) + 5(1) = -1005$$

$$\therefore \quad I_2 = \frac{\Delta_2}{\Delta} = \frac{-30}{1200} = -0.025$$

$$I_3 = \frac{\Delta_3}{\Delta} = \frac{-1005}{1200} = -0.8375$$

$\therefore$ The current through $R = 2\Omega$;

$$I_{2\Omega} = I_2 - I_3$$
$$= (-0.025) - (-0.8375)$$
$$I_{2\Omega} = 0.8125 \text{ A} \quad \text{ans.}$$

$\Rightarrow$ Voltage across 2Ω resistor;

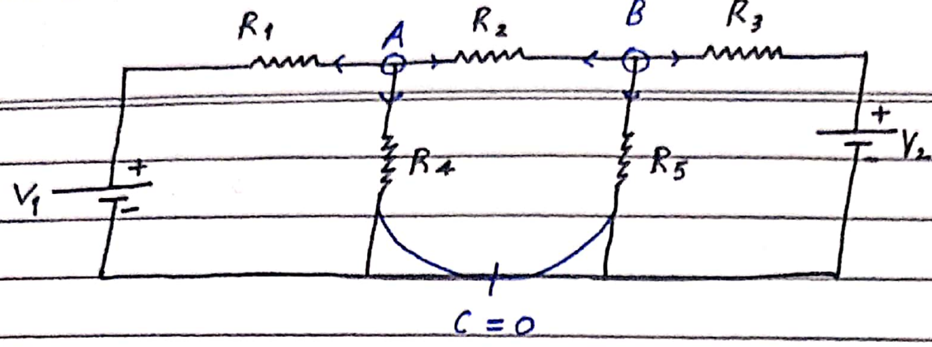
$$V_{2\Omega} = I_{2\Omega} \times 2$$
$$= 0.8125 \times 2$$
$$V_{2\Omega} = 1.625 \text{ V} \quad \text{Ans.}$$

Nodal Analysis (Based on KCL)

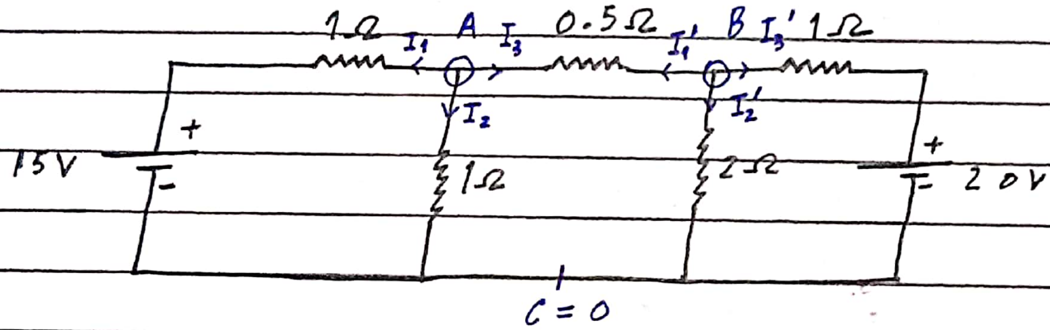
Nodal Analysis is based on KCL, in this method we define the voltage across each node as an independent.

For the application every node junction in the network where three or more branches meet is regarded as a node. One of these is regarded as a reference node or zero potential node.

Hence, the no. of simultaneous equations to be solved becomes (n-1) where n is the no. of independent nodes.



Q. Find the branch currents of the given network using nodal analysis.



Let 'A' and 'B' be two node junctions and 'C' be the reference node.

Applying KCL for node A

$$\frac{V_A - 15}{1} + \frac{V_A - 0}{1} + \frac{V_A - V_B}{0.5} = 0$$

$\therefore$ $2V_A - 15 + 2V_A - 2V_B = 0$

$4V_A - 2V_B = 15$ —— (1)

Now, applying KCL for node B

$$\frac{V_B - 20}{1} + \frac{V_B - 0}{2} + \frac{V_B - V_A}{0.5} = 0$$

$\frac{3}{2}V_B - 20 + 2V_B - 2V_A = 0$

$7V_B - 4V_A = 20 \times 2$

$7V_B - 4V_A = 40$ —— (2)

Adding ① and ②

$\therefore$ $4V_A - 2V_B = 15$
$7V_B - 4V_A = 40$

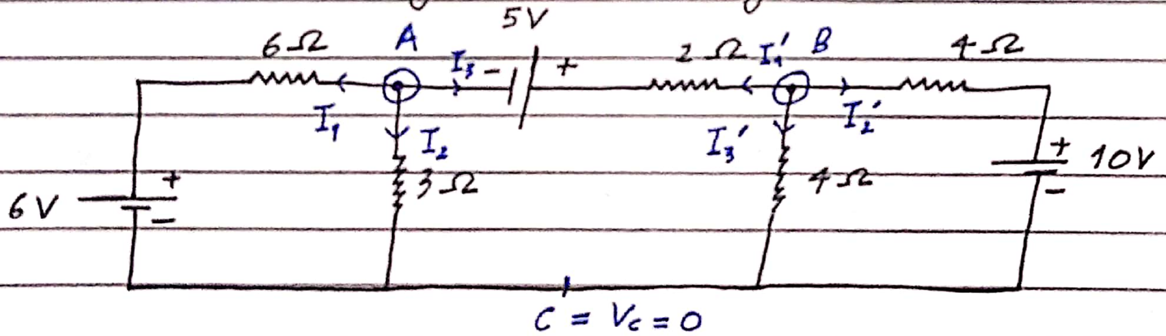
$5V_B = 55$

$\therefore$ $V_B = \frac{55}{5} = 11\,V$

$\Rightarrow$ $4V_A - 22 = 15$
$V_A = 9.25\,V$

$\therefore$ $I_1 = 9.25 - 15 = -5.75\,A$
$I_2 = 9.25\,A$
$I_3 = (9.25 - 11)2 = -3.5\,A$
$I_2' = 5.5\,A$
$I_3' = 11 - 20 = -9\,A$
$I_1' = (11 - 9.25) \times 2 = 3.5$
$I_{0.5\Omega} = 3.5 - (-3.5) = 7\,A$ Ans.

Q. Find the branch current in the given network using nodal analysis.



Let 'A' and 'B' be two node junctions with potential 'V_A' and 'V_B' respectively and 'C' be reference potential node.

$\therefore$ Applying KCL on node (A)

$$\frac{V_A - 6}{6} + \frac{V_A - 0}{3} + \frac{V_A + 5 - V_B}{2} = 0$$

$$V_A - 6 + 2V_A + 3V_A + 15 - 3V_B = 0$$
$$6V_A - 3V_B = -9$$
$$2V_A - V_B = -3 \quad \text{——— (1)}$$

Now, applying KCL on node 'B';

$$\frac{V_B - 5 - V_A}{2} + \frac{V_B}{4} + \frac{V_B - 10}{4} = 0$$

$$2V_B - 10 - 2V_A + 2V_B - 10 = 0$$
$$4V_B - 2V_A = 20 \quad \text{——— (2)}$$

$\therefore$ Adding (1) and (2)

$$\cancel{2V_A} - V_B = -3$$
$$4V_B - \cancel{2V_A} = 20$$
$$3V_B = 17$$

$$V_B = \frac{17}{3} = 5.66\ V$$

$$\therefore \quad 2V_A = -3 + V_B$$
$$V_A = \frac{-3 + 5.66}{2} = 1.33\ V$$

$$\Rightarrow \quad I_1 = \frac{V_A - 6}{6} = \frac{1.33 - 6}{6} = -0.77\ A$$

$$I_2 = \frac{V_A}{3} = \frac{1.33}{3} = 0.44\ A$$

$$I_3 = \frac{V_A + 5 - V_B}{2} = \frac{1.33 + 5 - 5.66}{2} = 0.335\ A$$

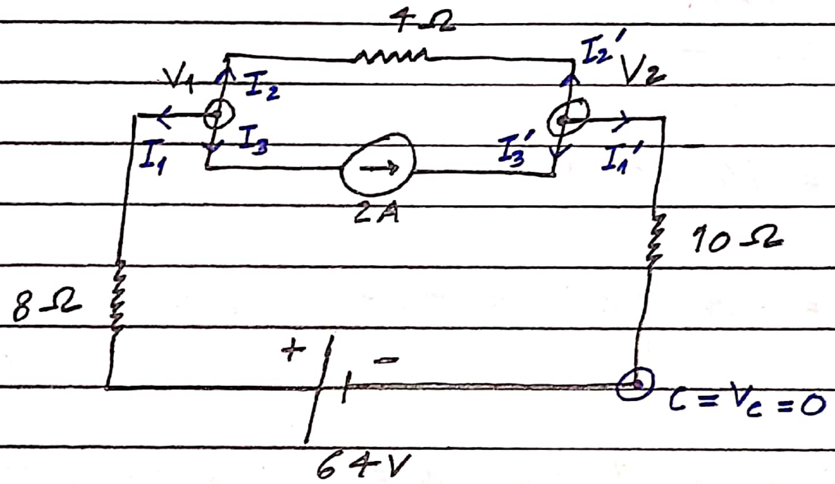
$$I_1' = \frac{V_B - 5 - V_A}{2} = \frac{5.66 - 5 - 1.33}{2} = -0.335$$

$$I_2' = \frac{V_B - 10}{4} = \frac{5.66 - 10}{4} = -1.085\,A$$

$$I_3' = \frac{V_B}{4} = \frac{5.66}{4} = 1.415\ A$$

$$\therefore\ I_{AB} = I_1 - I_1' = 0.335 - (-0.355) = 0.67\ \text{Ans.}$$

Q. Using Nodal Analysis, calculate V_1 and V_2.



Let 'C' be the reference node.

$2\,|\,10, 4$
$2\,|\,5$
5

Applying KCL on V_1.

$$\frac{V_1 - 64}{8} + \frac{V_1 - V_2}{4} + 2 = 0$$

$$V_1 - 64 + 2V_1 - 2V_2 = -16$$

$$3V_1 - 2V_2 = 48 \quad \text{——— (1)}$$

Applying KCL on V_2.

$$\frac{V_2}{10} + \frac{V_2 - V_1}{4} - 2 = 0$$

$$2V_2 + 5V_2 - 5V_1 = 40$$

$$7V_2 - 5V_1 = 40 \quad \text{——— (2)}$$

Multiplying eq ① by 5

$15V_1 - 10V_2 = 240$ —— ③

Multiplying eq ② by 3

$21V_2 - 15V_1 = 120$ —— ④

Adding ③ and ④

$$\cancel{15V_1} - 10V_2 = 240$$
$$21V_2 - \cancel{15V_1} = 120$$
$$11V_2 = 120$$

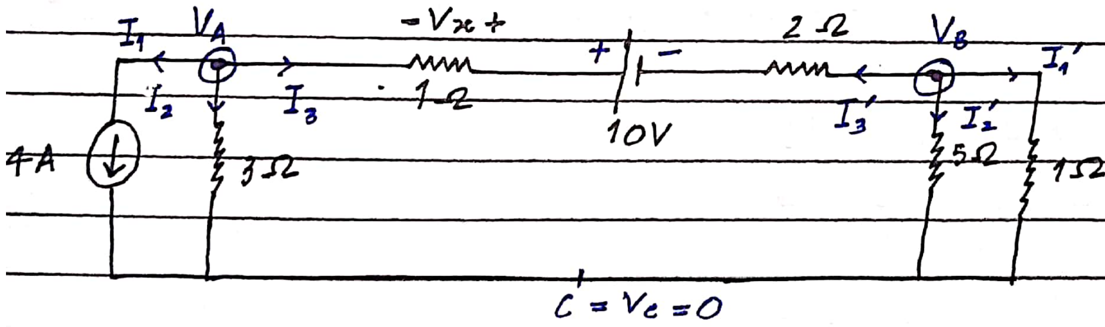
$$V_2 = 10.90 \text{ V}$$

and

$$V_1 = \frac{48+2V_2}{3} = \frac{48+21.81}{3} = 23.27\text{V}$$

Ans.

Q. Calculate the current through 5Ω also find V_x.



Let 'A' and 'B' be two nodes with potentials as 'V_A' and 'V_B' respectively.
'C' be the reference node.

Applying KCL on node 'A'

$$4 + \frac{V_A}{3} + \frac{V_A - 10 - V_B}{3} = 0$$

$$2V_A - V_B = -2 \quad \text{—— ①}$$

Applying KCL on node 'B'

$\therefore \quad V_B + \frac{V_B}{5} + \frac{V_B + 10 - V_A}{3} = 0$

$15 V_B + 3 V_B + 5 V_B + 50 - 5 V_A = 0$

$23 V_B - 5 V_A = -50 \quad — (2)$

$\therefore \quad (1) \times 5 + (2) \times 2$

$\Rightarrow \quad 10 V_A - 5 V_B = -10$

$46 V_B - 10 V_A = -100$

$41 V_B = -110$

$V_B = \frac{-110}{41} = -2.68 V$

$\therefore \quad V_A = \frac{-2 + V_B}{2} = -2.34 V$

$\therefore \quad I_{3\Omega} = \frac{V_B}{5} = \frac{-2.68}{5} = -0.536 A$ ans.

and $I_3 = \frac{V_A - 10 - V_B}{3} = \frac{-2.34 - 10 + 2.68}{3} = -3.22 A$

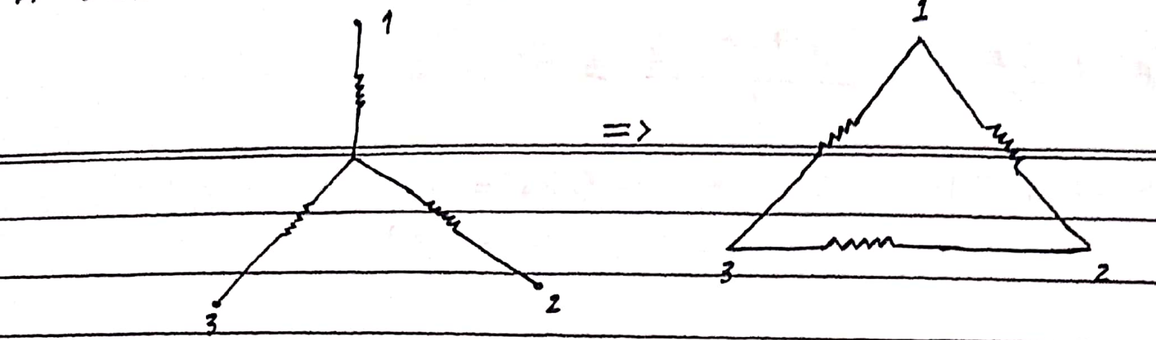
$I_3' = \frac{V_B + 10 - V_A}{3} = \frac{-2.68 + 10 + 2.3}{3} = 3.22 A$

$I_{1\Omega} = I_3' - I_3$

$\therefore \quad I_{1\Omega} = 3.22 - (-3.22) = 6.44 A$

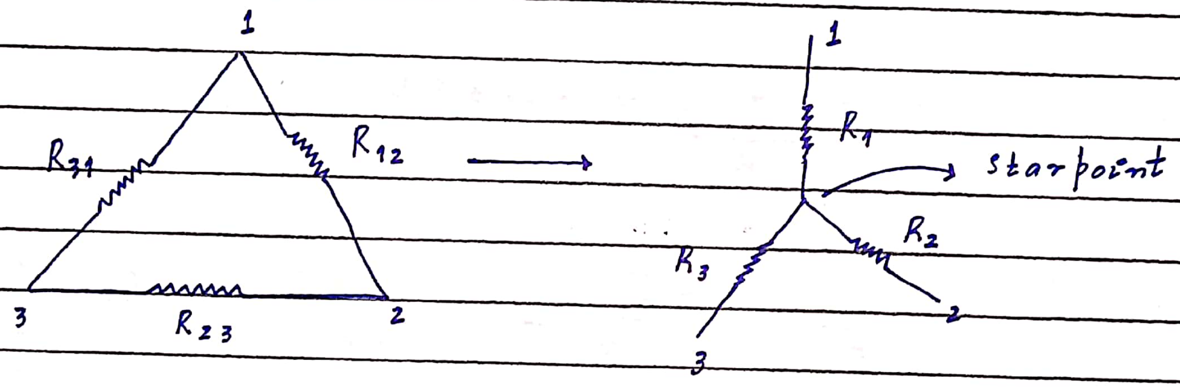
$\Rightarrow \quad V_x = 1 \times 6.44 = 6.44 V$ Ans.

Star-Delta Transformation



There are some networks in which resistors are neither connected in series nor in parallel such as star-delta networks.

In such simulations, it is not possible to simplify the network by series and parallel circuit nodes however such network can be simplify by using star-delta transformation technique.



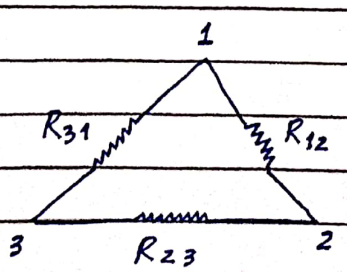
→ Delta to star transformation.

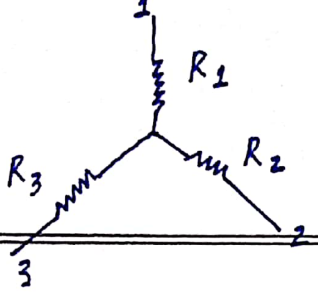
Consider three resistors R_{12}, R_{23} and R_{31} connected in Δ, these three given resistors can be replaced by three resistors R_1, R_2 and R_3 connected in star.

First we take delta connection between terminal 1 and 2, there are two parallel path one having resistance R_{12} and other having resistance $R_{23} + R_{31}$. Therefore, the equivalent resistance between one and two:

$$R_{12} \parallel (R_{31} + R_{23})$$

$$\therefore \quad R_{eq} = \frac{R_{12}(R_{31}+R_{23})}{R_{12}+R_{31}+R_{23}}$$





Now, we take star connection, The resistance between same terminal is equal to $R_1 + R_2$.

Hence, as the terminal resistance had to be same i.e resistance between 1 and 2 for star is equal to 1 and 2 for delta.

$$\Rightarrow \quad R_1 + R_2 = \frac{R_{12}(R_{23} + R_{31})}{R_{12} + R_{23} + R_{31}} \quad \text{—— (1)}$$

Similarly,

$$R_2 + R_3 = \frac{R_{23}(R_{31} + R_{12})}{R_{12} + R_{23} + R_{31}} \quad \text{— (2)}$$

and

$$R_1 + R_3 = \frac{R_{31}(R_{12} + R_{23})}{R_{12} + R_{23} + R_{31}} \quad \text{—— (3)}$$

Now, subtracting (2) from (1)

$$\therefore \quad R_1 - R_3 = \frac{R_{12}R_{23} + R_{12}R_{31} - R_{23}R_{31} - R_{23}R_{12}}{R_{12} + R_{23} + R_{31}}$$

$$R_1 - R_3 = \frac{R_{12}R_{31} - R_{23}R_{31}}{R_{12} + R_{23} + R_{31}}$$

Adding the above result in eq (3)

$$\therefore \quad 2R_1 = \frac{R_{12}R_{31} + R_{12}R_{31}}{R_{12} + R_{23} + R_{31}}$$

$$\therefore \quad R_1 = \frac{R_{12}R_{31}}{R_{12} + R_{23} + R_{31}} \quad \text{— (4)}$$

Similarly: $R_2 = \dfrac{R_{12} \cdot R_{23}}{R_{12} + R_{23} + R_{31}}$ (5) and $R_3 = \dfrac{R_{31} \cdot R_{23}}{R_{12} + R_{23} + R_{31}}$ (6)

⇒ Star to Delta transformation

Dividing eq ④ and ⑤

$$\therefore \frac{R_1}{R_2} = \frac{R_{12} \cdot R_{31}}{R_{12} \cdot R_{23}}$$

$$\frac{R_1}{R_2} = \frac{R_{31}}{R_{23}}$$

$$\therefore R_{31} = \frac{R_1 \times R_{23}}{R_2}$$

and Dividing eq ④ and ⑥

$$\frac{R_1}{R_3} = \frac{R_{12}}{R_{23}}$$

$$\therefore R_{12} = \frac{R_1 \cdot R_{23}}{R_3}$$

Subsituting the values of R_{12} and R_{31} in ④

$$R_1 = \frac{\frac{R_1 \cdot R_{23}}{R_3} \times \frac{R_1 \cdot R_{23}}{R_2}}{(R_{12} + R_{23} + R_{31})}$$

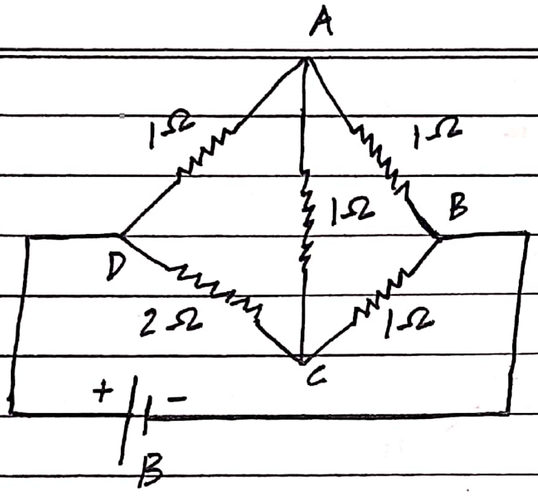
$$\therefore R_{23} = R_2 + R_3 + \frac{R_2 R_3}{R_1}$$

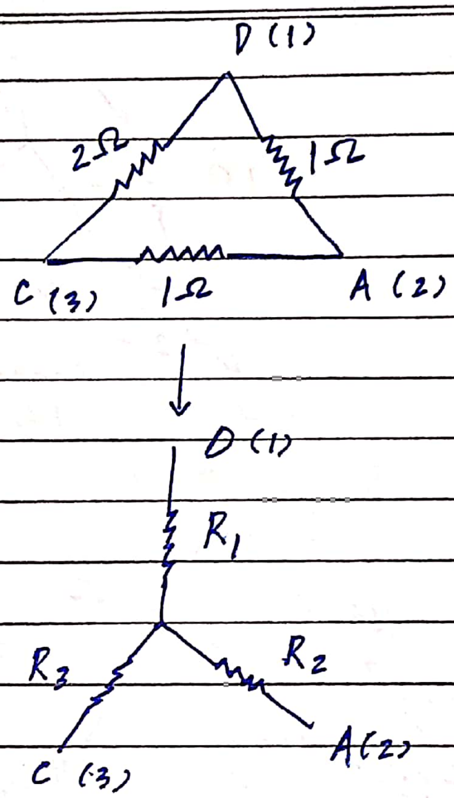
$$R_{12} = R_1 + R_2 + \frac{R_1 R_2}{R_3}$$

$$R_{31} = R_3 + R_1 + \frac{R_1 R_3}{R_2}$$

Ans.

Q. Find the equivalent resistance across the battery terminal of the given network using Star to Delta and Delta to Star transformation technique.



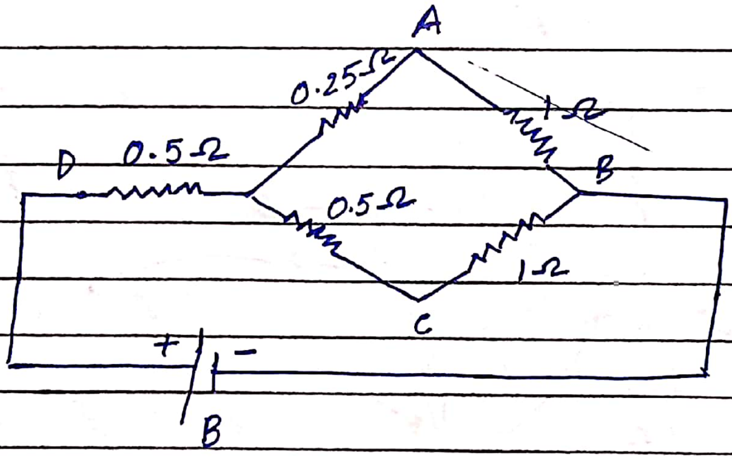


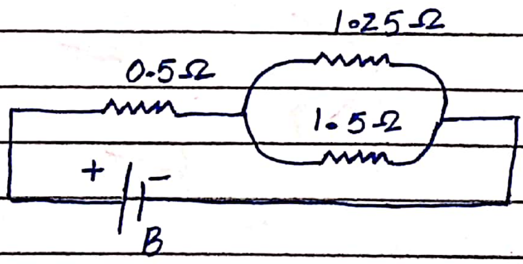
$\therefore \quad R_1 = \dfrac{1 \times 2}{4} = \dfrac{1}{2} = 0.5\ \Omega$

$R_2 = \dfrac{1 \times 1}{4} = 0.25\ \Omega$

$R_3 = \dfrac{1 \times 2}{4} = 0.5\ \Omega$

$\therefore$

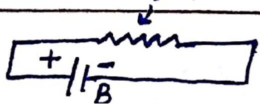




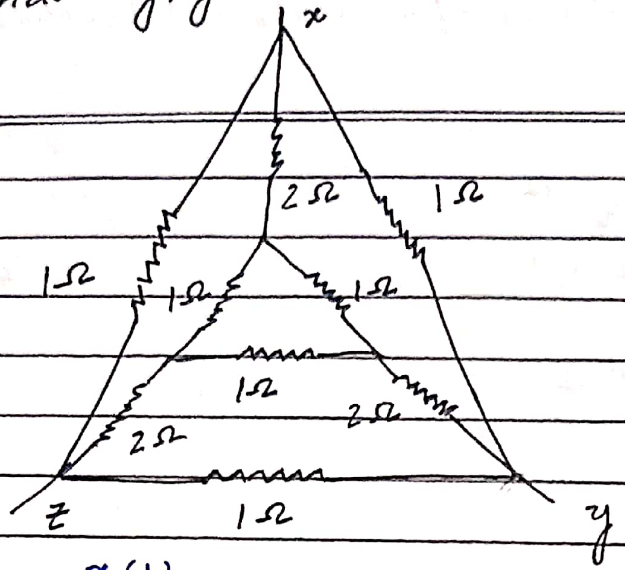
$\therefore \quad R_{eq} = 0.5 + (1.25 \,//\, 1.5)$

$R_{eq} = 1.18\ \Omega$

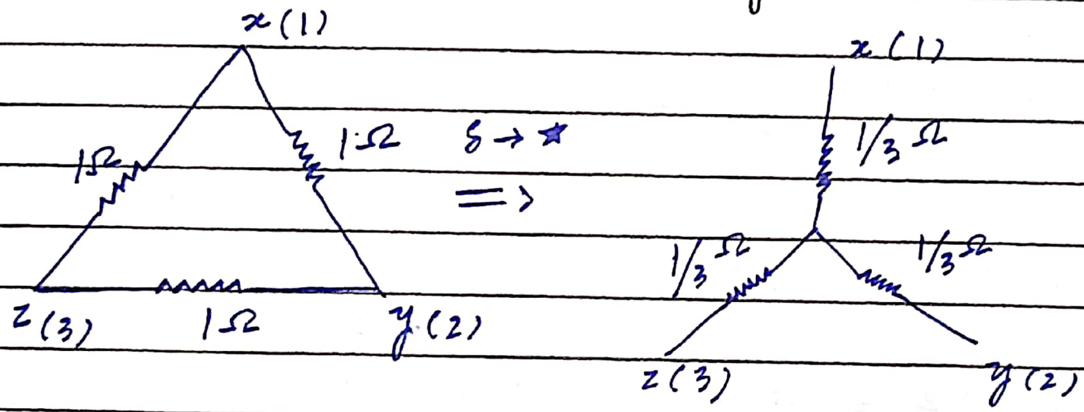
$\therefore$



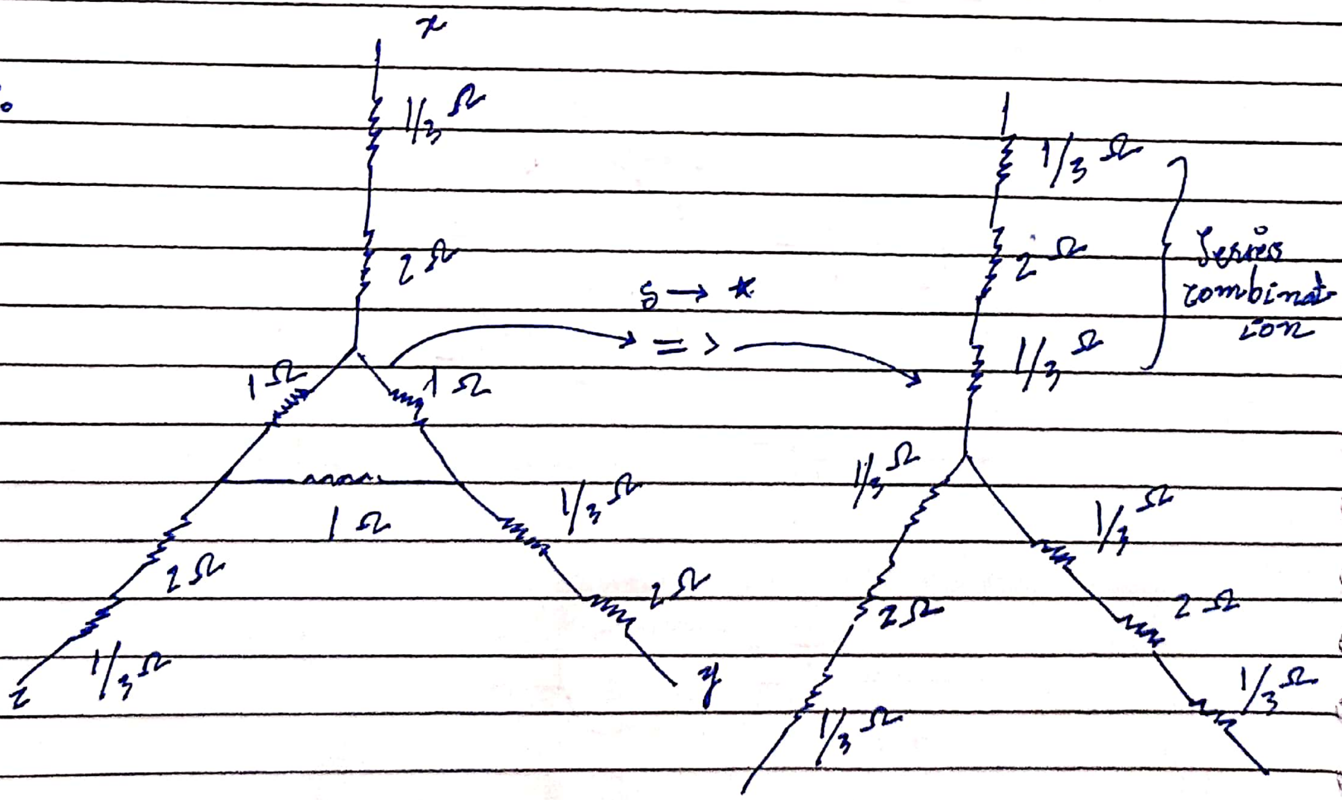
Q. Determine the equivalent resistance between the terminal xy, yz and zx.



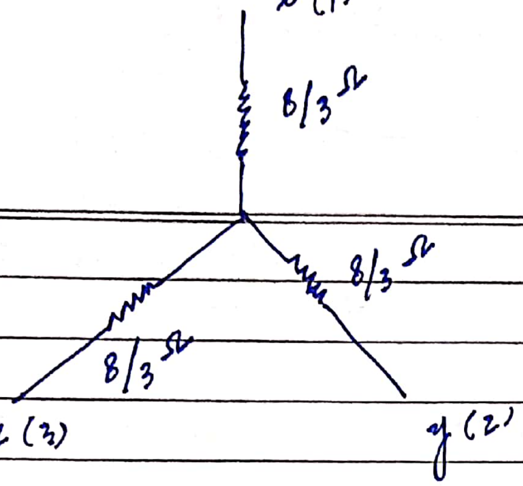
∴



∴



$\therefore$



$\frac{2}{3}+2$

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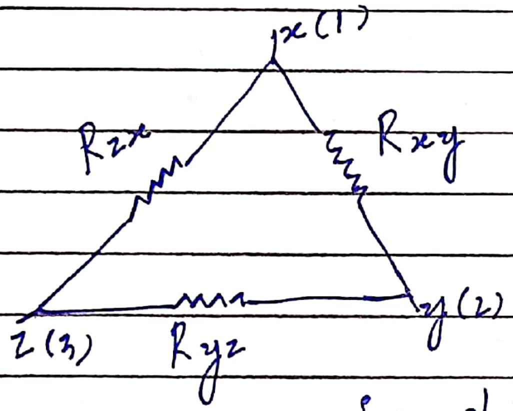
کو بیٹھے
سر پے

Y → Δ

Delta transformation



$$\therefore R_{xy} = R_x + R_y + \frac{R_x R_y}{R_z}$$

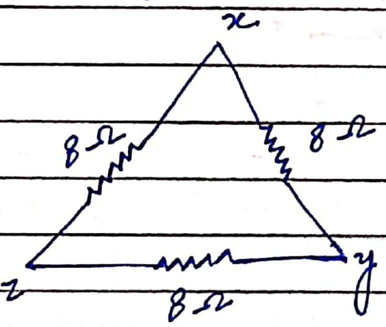
$$= \frac{8}{3} + \frac{8}{3} + \frac{(8/3)(8/3)}{(8/3)}$$

$$R_{xy} = \frac{8}{3} \times 3 = 8\,\Omega$$

Similarly ; $R_{yz} = 8\,\Omega$

$R_{zx} = 8\,\Omega$

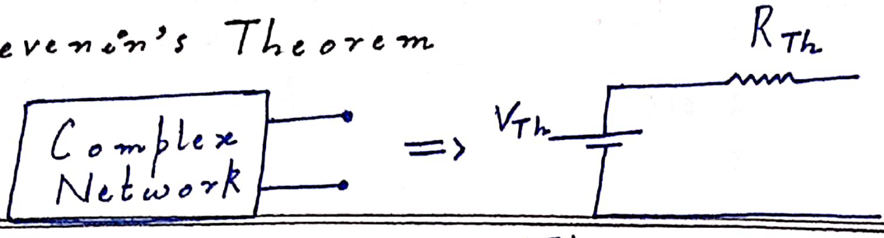
$\therefore$ The given circuit transforms into :



Hence,
the equivalent resistance between terminals xy, yz and zx is 8 Ω.

Ans.

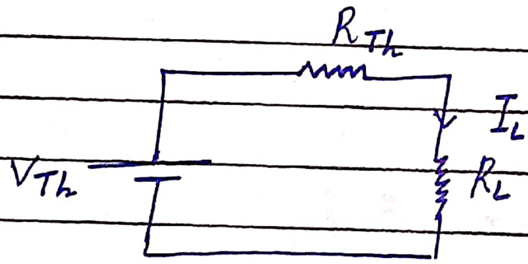
Thevenin's Theorem



Thevenin's Equivalent Network.

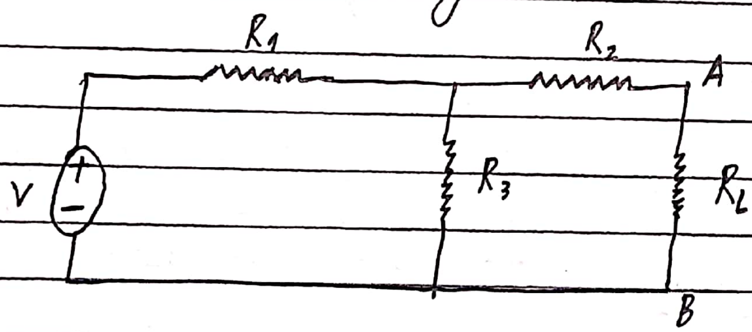
Thevenin's Theorem is the most important theorem that can be used for simplification of complicated network.

Any linear bilateral network having two terminals can be replaced by an equivalent network or circuit. Consisting a single voltage source V_{Th} in series with single resistance R_{Th}.



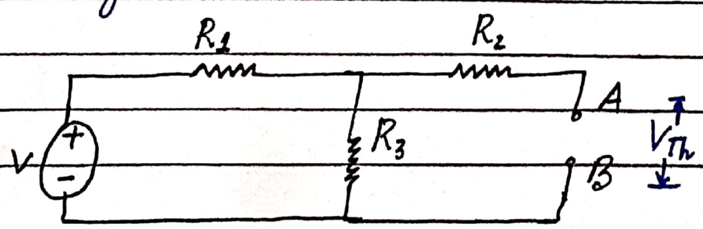
$$I_L = \frac{V_{Th}}{R_{Th} + R_L}$$

→ How to Thevenize a given network



let us consider a complicated network, we are to find current flowing through R_L by using Thevenin's Theorem.

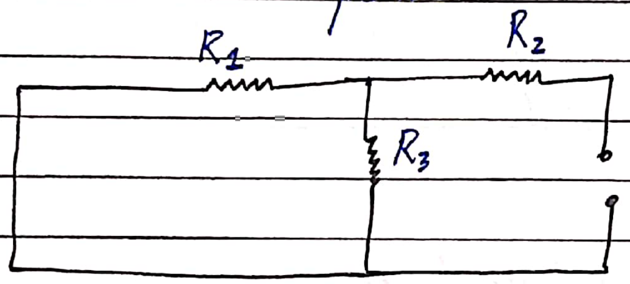
Step 1: Remove the load resistor and calculate the Thevenin's equivalent voltage V_{Th} across AB



$\therefore$ $V_{Th} = IR_3 =$ voltage across R_3

$$V_{Th} = \frac{V}{R_1 + R_3} \cdot R_3$$

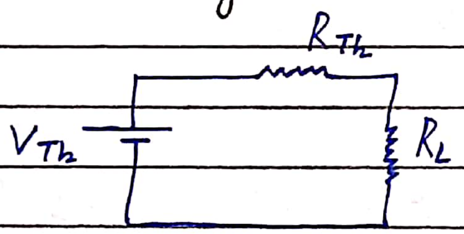
Step 2: Calculate R_{th} by voltage source shorted and current source open.



$$R_{Th} = (R_1 \parallel R_3) + R_2$$

$$R_{Th} = \frac{R_1 R_3}{R_1 + R_3} + R_2 \ \Omega.$$

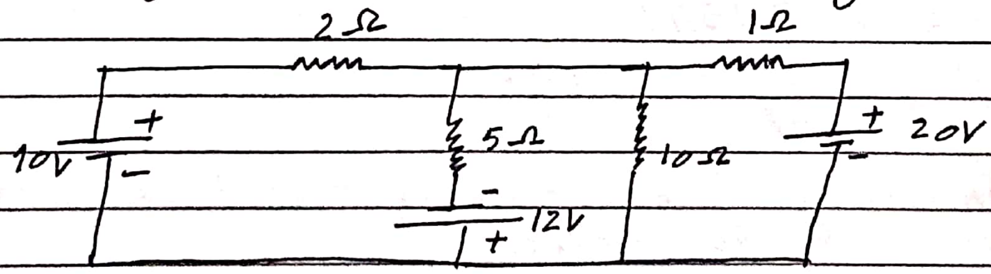
Step 3: Draw the Thevenin's equivalent network by connecting V_{Th} and R_{Th} in series.



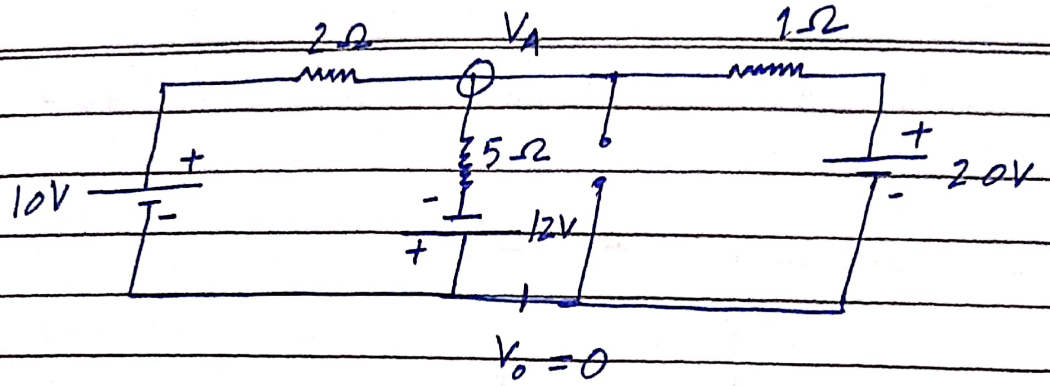
$$\boxed{I_L = \frac{V_{Th}}{R_{Th} + R_L}}$$

NOTE: Now R_L is connected back across the terminal AB and find the load current I_L.

Q. Find the current through 10Ω resistance utilizing Thevenin's theorem of the given network.



Removing $R_L = 10\Omega$ from the given circuit and calculating V_{Th}.



∴ Applying KCL at node A

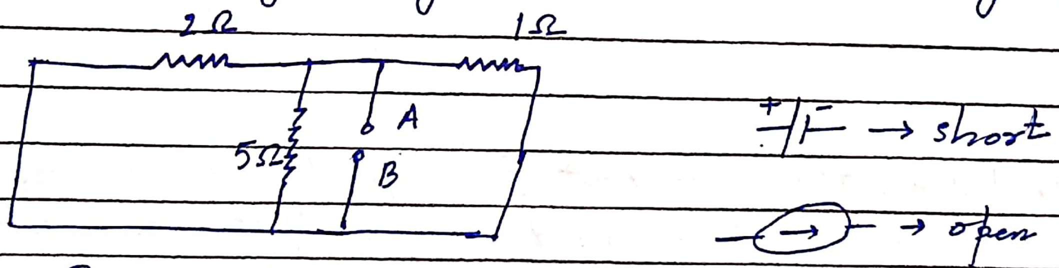
$$\frac{V_A - 10}{2} + \frac{V_A + 12}{5} + \frac{V_A - 20}{1} = 0$$

$$5V_A - 50 + 2V_A + 24 + 10V_A - 200 = 0$$

$$17 V_A = 226$$

$$V_A = \frac{226}{17} = 13.29\,V$$

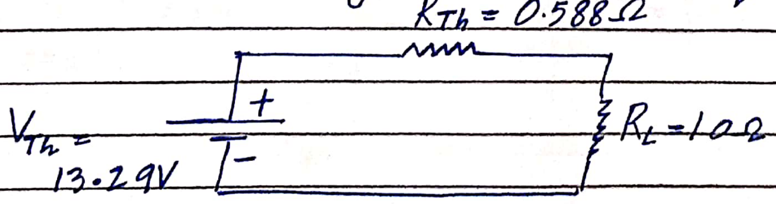
→ Now, shorting voltage sources and calculating R_{Th}



∴ $R_{Th} = 2 \| 5 \| 1$

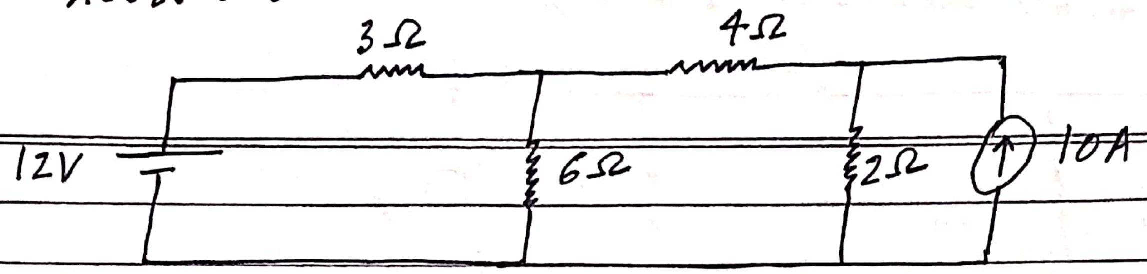
$$R_{Th} = \frac{10}{17} = 0.588\,\Omega$$

Now, drawing Thevenin's equivalent circuit:

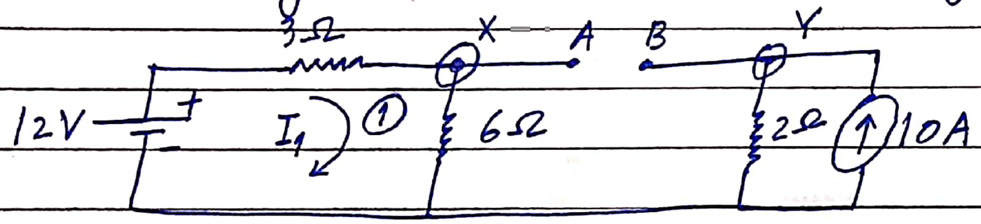


∴ $I_L = \frac{13.29}{0.588 + 10} = \frac{13.29}{10.588} = 1.25\,A$ Ans.

Q. Calculate the current flowing through 4Ω resistor.



Removing $R_L = 4\Omega$ and calculating V_{Th}.



$\therefore \quad I_1 = \frac{12}{3+6} = \frac{12}{9} = \frac{4}{3}$ A

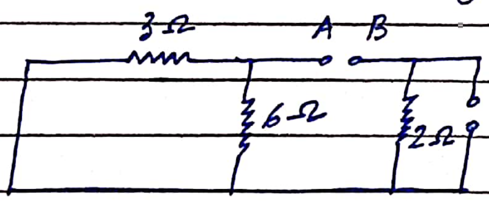
$\therefore$ Voltage dropped across $3\Omega = 3 \times 4/3 = 4V$

Hence, voltage on node $X = 12 - 4 = 8V$
and voltage on node $Y = 10 \times 2 = 20V$

$\Rightarrow \quad V_{AB} = V_{XY} = 20 - 8 = 12V = V_{Th}$.

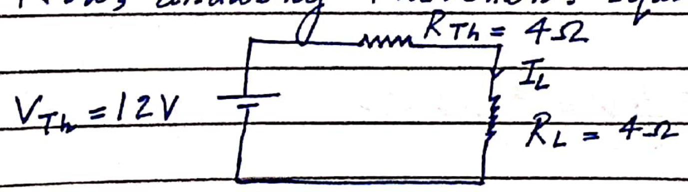
Now, shorting voltage sources and opening current sources and calculating R_{Th}.

$\Rightarrow$



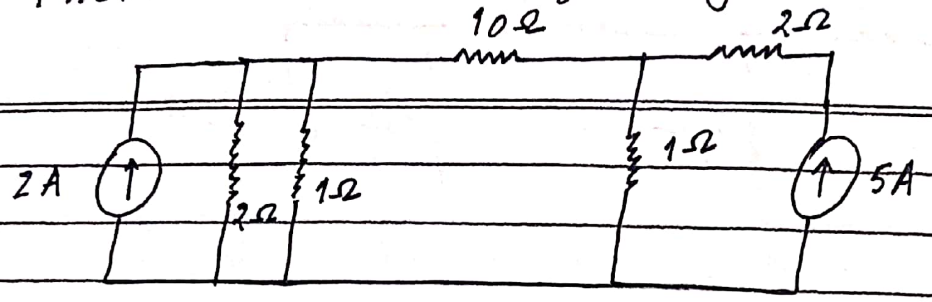
$\therefore \quad R_{Th} = (3 \| 6) + 2 = 4\Omega$

Now, drawing Thevenin's Equivalent circuit:

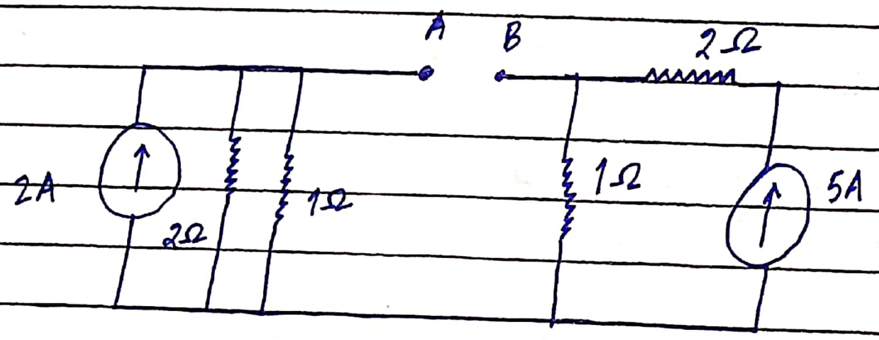


Hence,
$I_L = \frac{12}{4+4} = 1.5$ Ans.

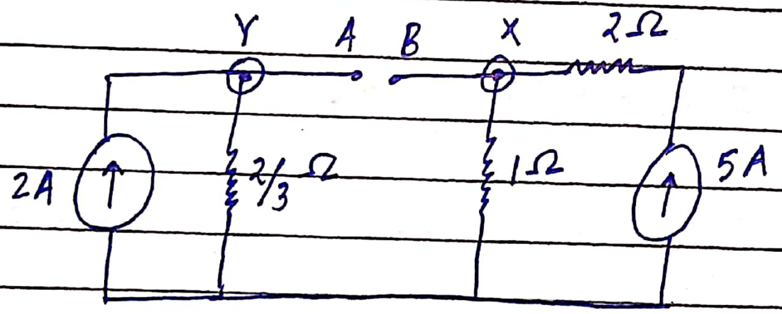
Q. Find the power loss in 10Ω resistor utilizing Thevenin's theorem of the given circuit.



Removing $R_L = 10\Omega$ and calculating V_{Th}.



∴ Replacing 2Ω and 1Ω with their equivalent resistance of $\frac{2}{3}\Omega$ (Parallel combination).



∴ Voltage on $Y = 2 \times \frac{2}{3} = \frac{4}{3} V$

∴ Voltage across $2\Omega = 5 \times 2 = 10V$

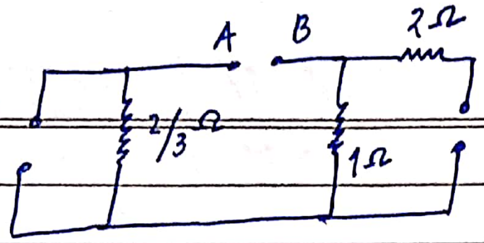
Voltage across $1\Omega = 5 \times 1 = 5V$

∴ Voltage on $X = 10 - 5 = 5V$

Now,

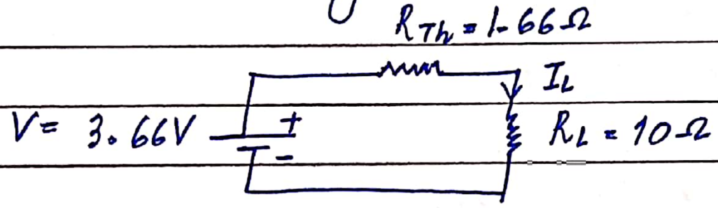
$V_{Th} = V_{AB} = 5 - \frac{4}{3} = \frac{11}{3} V = 3.66 V.$

Now, shorting voltage sources and opening current sources, and calculating R_{Th}.



$$\therefore \quad R_{Th} = 1 + \frac{2}{3} = \frac{5}{3} = 1.66\,\Omega.$$

Now, drawing Thevenin's equivalent circuit.

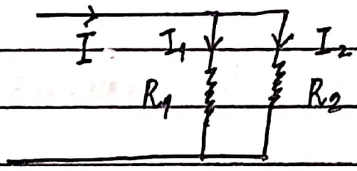


$$\therefore \quad I_L = \frac{3.66}{1.66+10} = \frac{3.66}{11.66} = 0.31\,A$$

$\Rightarrow$ Power dissipated in $10\Omega = I^2R = (0.31)^2 \times 10$

$= 0.961\,W$ Ans.

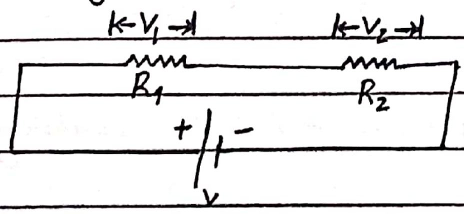
→ Current Divider Rule



$$I_1 = \frac{I \cdot R_2}{R_1 + R_2}$$

$$I_2 = \frac{I \cdot R_1}{R_1 + R_2}$$

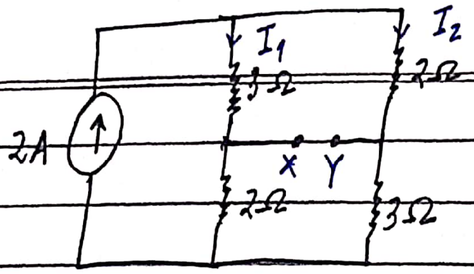
→ Voltage Divider Rule



$$V_1 = \frac{R_1 \cdot V}{R_1 + R_2}$$

$$V_2 = \frac{R_2 \cdot V}{R_1 + R_2}$$

Q. Find the Thevenin's equivalent network across the terminal xy in the given terminal.



$I_1 = 1A$
$I_2 = 1A$

[Because both arms have same resistance, hence current divides equally]

$$\Rightarrow \quad V_X = 3 \times I_1 = 3\,V$$

$$V_Y = 2 \times I_2 = 2\,V$$

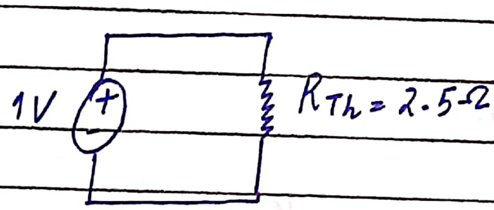
$$\therefore \quad V_{XY} = 3 - 2 = 1V = V_{Th}$$

and

$$R_{Th} = (3 \parallel 2) + (3 \parallel 2)$$

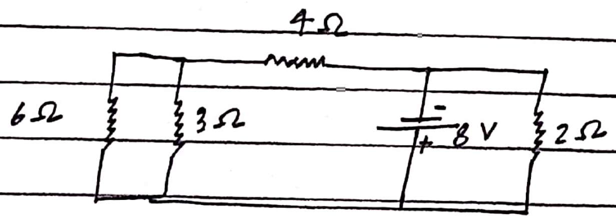
$$= \frac{6}{5} + \frac{6}{5} = \frac{12}{5} = 2.5\,\Omega$$

$\therefore$ Drawing Thevenin's equivalent network

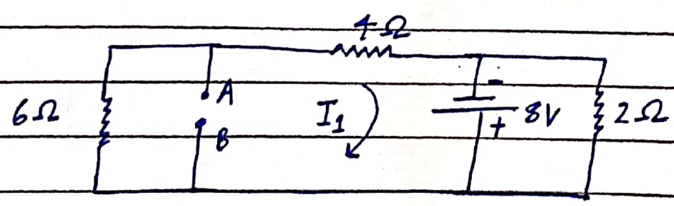


Ans.

Q. Using Thevenin's Theorem, calculate the current through 3Ω resistor.



Removing the load resistor $R_L = 3\Omega$ and calculating V_{Th}.

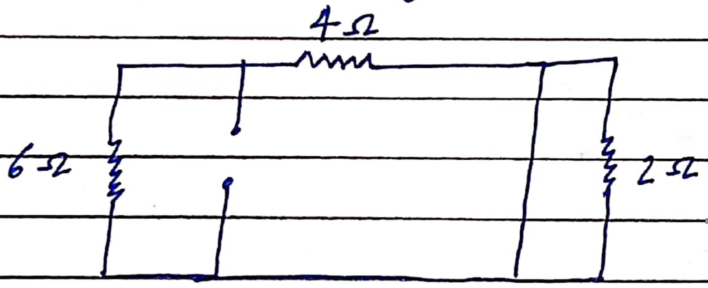


∴ Calculating I_1 from mesh 1

$$10 I_1 = 8$$
$$I_1 = 0.8 A$$

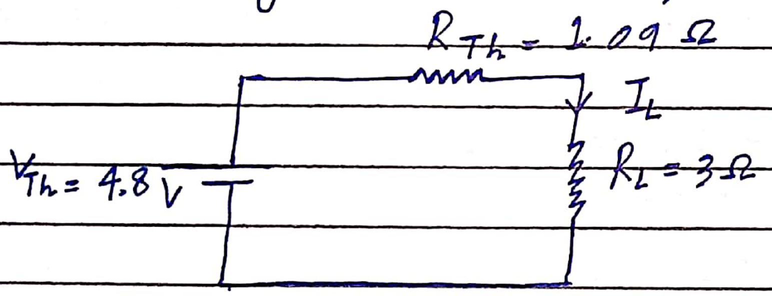
∴ Voltage across $6\Omega = 6 \times 0.8 = 4.8V$
This is equal to the voltage across AB i.e V_{th} as AB is parallel to 6Ω.

Now, calculating R_{Th}



∴ $R_{Th} = (6 \| 4) \| 2$
$= (2.4\Omega) \| (2\Omega)$
$= 1.09\Omega$

Drawing Thevenin's Equivalent circuit ;

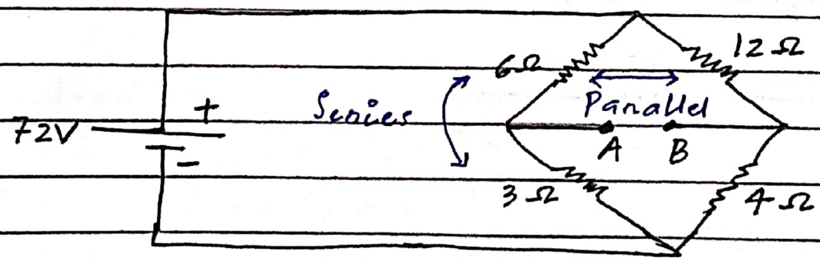


$$\Rightarrow \quad I_L = \frac{4.8}{1.09+3} = 1.173 A$$

Ans.

इस सवाल मे गड़बड़ हो सकती है। बेहतर है के आप देख ले ~~एक~~ एक बार।

Q. Find the Thevenin's equivalent network across the terminal AB after given network;



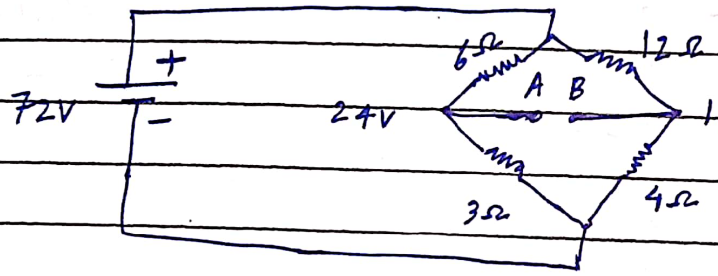
Applying voltage divider for 6Ω resistor

$\therefore V_{6\Omega} = \frac{6 \times 72}{6+3} = 48V$ (6 and 3Ω resistors are in series)

Similarly for 12Ω resistor

$\therefore V_{12} = \frac{12 \times 72}{12+4} = 54V$ (12 and 4Ω resistors are in series).

Now;

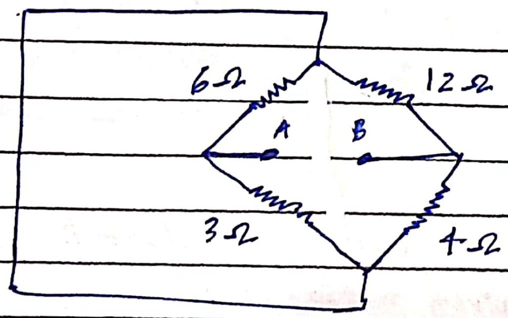


$\therefore V_A = 72 - 48 = 24V$

$V_B = 72 - 54 = 18V$

$\therefore V_{Th} = V_A - V_B = 24 - 18 = 6V$

Now, calculating R_{Th}

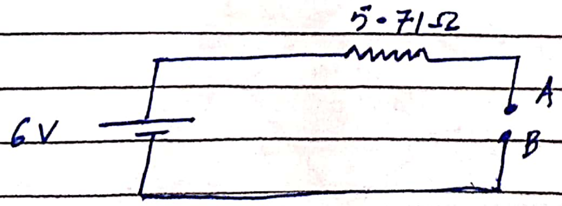


$R_{Th} = (6 \| 12) + (3 \| 4)$

$= 4 + 12/7$

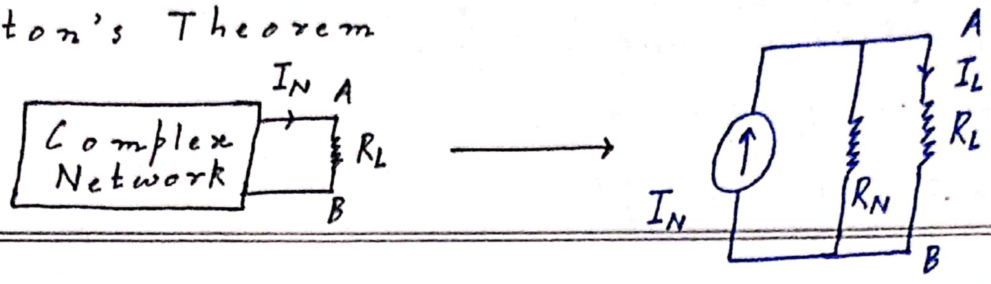
$R_{Th} = 40/7\ \Omega = 5.71\Omega$

$\therefore$ Thevenin's Equivalent Network;



Ans.

Norton's Theorem



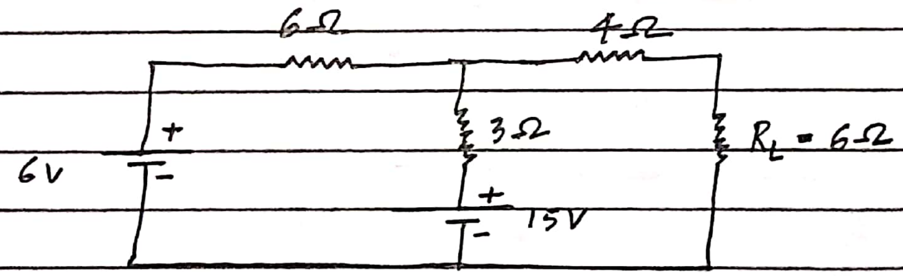
Norton equivalent circuit.

$$I_L = \frac{R_N I_N}{R_N + R_L}$$

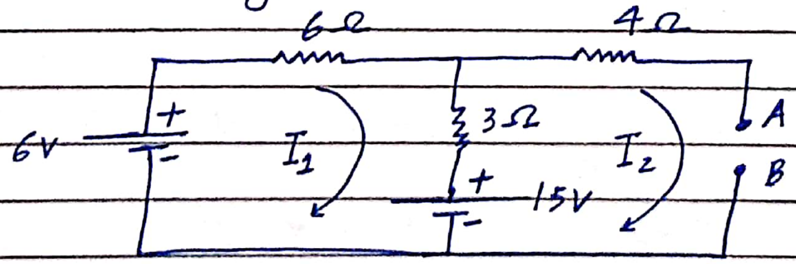
→ Norton Theorem is an alternative theorem to the Thevenin's theorem and applicable for a complex network. By using this theorem we can replace a complex network by a current in parallel with a single resistor.

Any linear bilateral network having a terminal A and B can be replaced by single current source I_N in parallel with a single resistor R_L.

Q. Find the current across $R_L = 6\Omega$, using Norton's Theorem.



Removing $R_L = 6\Omega$ and calculating I_N



Applying KVL in mesh 1

$$+6 - 6I_1 - 3(I_1 - I_2) - 15 = 0$$

$$-9I_1 + 3I_2 = 9 \quad \text{—— } (1)$$

Applying KVL in mesh (2)

$$15 - 3(I_2 - I_1) - 4I_2 = 0$$

$$-7I_2 + 3I_1 = -15 \text{ ——— } (2)$$

Solving (1) and (2)

$$-9I_1 + 3I_2 = 9$$

$$9I_1 - 2I_2 = -45$$

$$+ \quad + \quad +$$

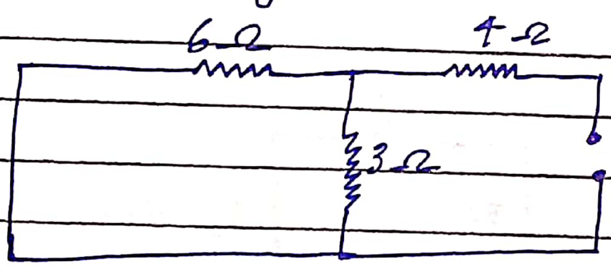
$$-18I_2 = -36$$

$$I_2 = \frac{36}{18} = 2A$$

And;

$$I_2 = I_N = 2A$$

Now, calculating R_N;

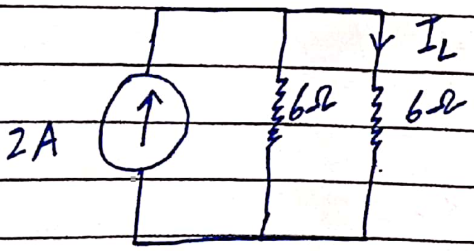


$$\therefore \quad R_N = (6 \| 3) + 4$$

$$= \frac{18}{9} + 4$$

$$R_N = 6\Omega \text{ Ans.}$$

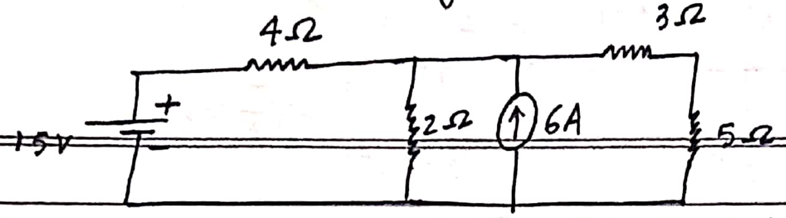
Drawing, Norton's equivalent circuit;



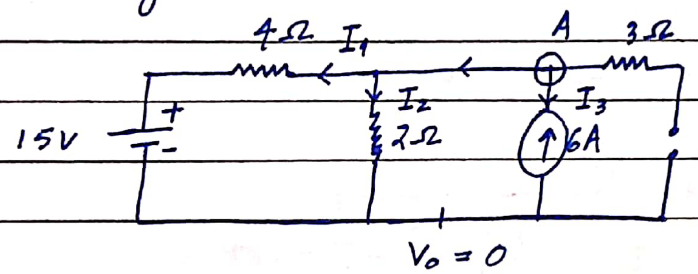
$$I_L = \frac{I_N R_N}{R_L + R_N}$$

$$I_L = \frac{6 \times 2}{12} = 1A \text{ ans.}$$

Q. Determine the current 5Ω resistor using Norton Theorem in given network.



Removing $R_L = 5\Omega$ and calculating I_N



Applying Nodal Analysis at 'A'

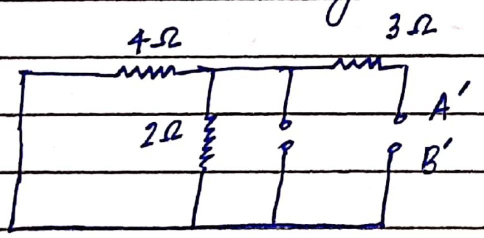
∴ Assuming I_1, I_2 and I_3 as outgoing current from node A.

=> Applying KCL.

$$\therefore \frac{V_A - 15}{4} + \frac{V_A}{2} - 6 = 0$$

$$\therefore \boxed{V_A = 13V} = V_N$$

Now, calculating R_N

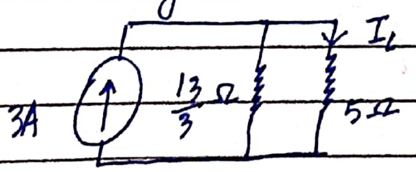


$$R_N = (4 // 2) + 3$$

$$R_N = 4/3 + 3 = \frac{13}{3}\Omega$$

$$\therefore I_N = \frac{V_N}{R_N} = \frac{13 \times 3}{13} = 3A$$

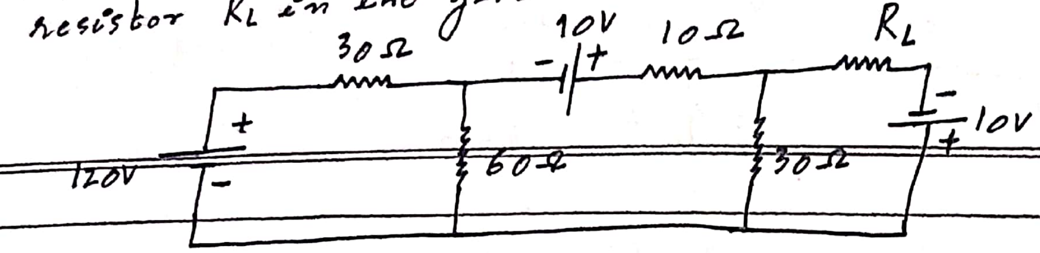
Now, drawing Norton's equivalent circuit;



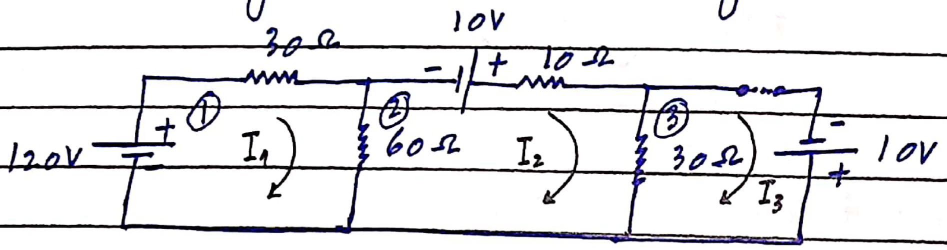
$$\Rightarrow I_L = \frac{R_N I_N}{R_N + R_L} = \frac{\frac{13}{3} \times 3}{\frac{13}{3} + 5} = 1.39A$$

Ans.

Q. Find the Norton equivalent network across the resistor R_L in the given network.



Removing R_L and calculating I_N ;



∴ KVL for mesh ①

$$120 - 30I_1 - 60I_1 + 60I_2 = 0$$
$$-90I_1 + 60I_2 = -120$$
$$-3I_1 + 2I_2 = -4 \quad \text{——— } ①$$

KVL for mesh ②

$$-60I_2 + 60I_1 + 10 - 10I_2 - 30I_2 + 30I_3 = 0$$
$$-100I_2 + 60I_1 + 30I_3 = -10$$
$$6I_1 - 10I_2 + 3I_3 = -1 \quad \text{——— } ②$$

KVL for mesh ③

$$-30I_3 + 30I_2 + 10 = 0$$
$$3I_2 - 3I_3 = -1 \quad \text{——— } ③$$

Arranging mesh equations into matrix form and using cramer's rule.

$$\begin{bmatrix} -3 & 2 & 0 \\ 6 & -10 & 3 \\ 0 & 3 & -3 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} -4 \\ -1 \\ -1 \end{bmatrix}$$

$$\therefore \Delta = -3(30-9) - 2(-18)$$
$$= -3(21) + 36$$
$$= -63 + 36$$
$$\Delta = -27$$

$$\Delta_3 = \begin{vmatrix} -3 & 2 & -4 \\ 6 & -10 & -1 \\ 0 & 3 & -1 \end{vmatrix}$$

$$= -3(10+3) - 2(-6) - 4(18)$$

$$= -39 + 12 - 72$$

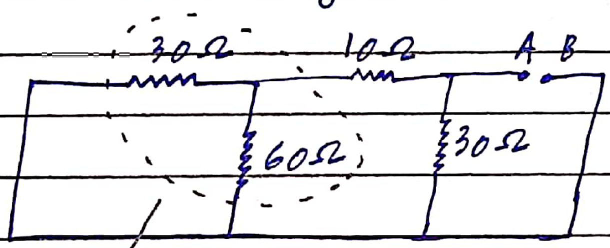
$$\Delta_3 = -99$$

$$\therefore \quad I_3 = \frac{\Delta_3}{\Delta} = \frac{-99}{-27} = \frac{11}{3} = 3.66\text{ A}$$

Ans.

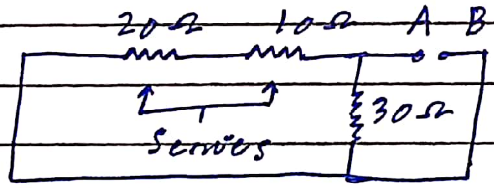
$$\Rightarrow \quad I_N = 3.66\text{ A}$$

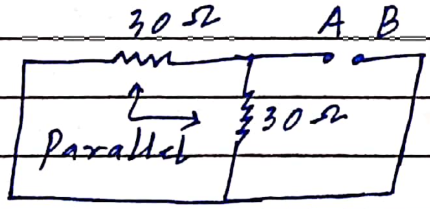
Now, calculating R_{Th}



Parallel combination

$R_{eq} = 20\Omega$

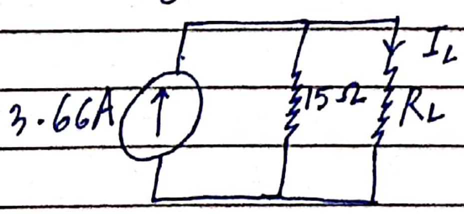




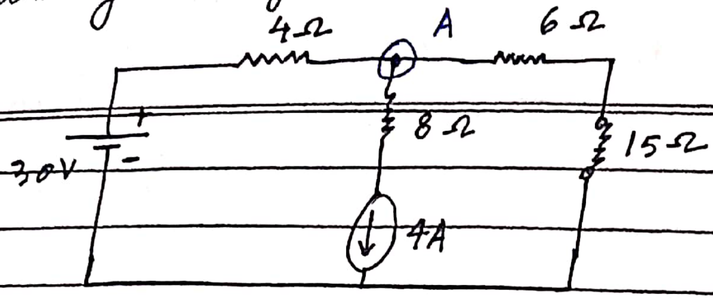
$\Rightarrow \quad R_N = 15\Omega$

$$R_{eq} = \frac{30 \times \cancel{30}}{\cancel{60}_{2}} = 15\Omega$$

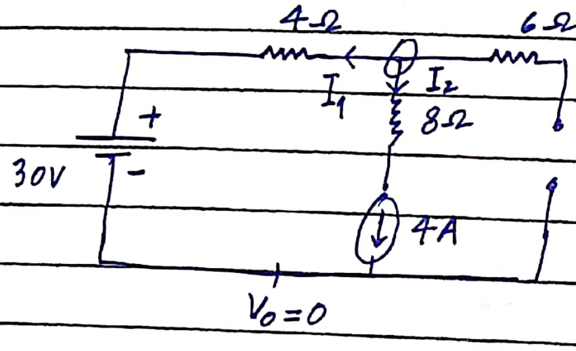
Now, drawing Norton's equivalent Network:



Q. Using Norton Theorem, evaluate the current flowing through 15 Ω resistor.



Removing $R_L = 15\Omega$ and applying KCL at 'A'

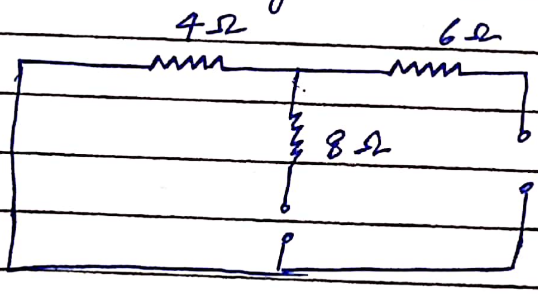


$$\therefore \frac{V_A - 30}{4} + 4 = 0$$

$$V_A - 30 = -16$$

$$V_A = 14V = V_{Th}$$

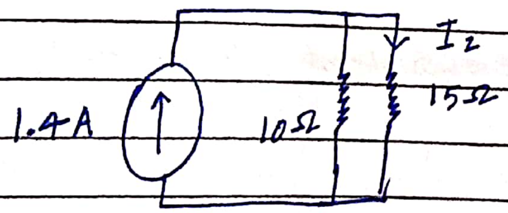
Now, calculating R_N:



$$R_N = 4 + 6 = 10\Omega$$

$$\Rightarrow I_N = \frac{V_N}{R_N} = \frac{14}{10} = 1.4A$$

Drawing: Norton's equivalent circuit ⇒



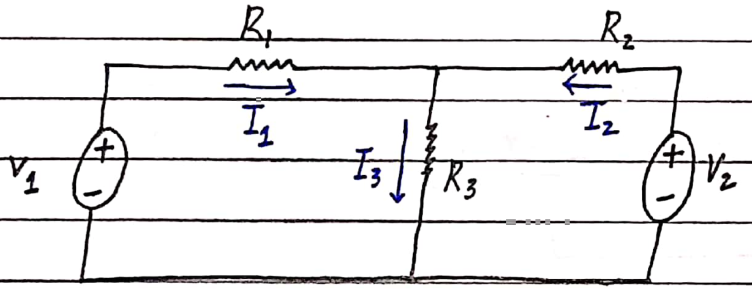
$$\therefore I_2 = \frac{1.4 \times 10}{10 + 15} = \frac{14}{25} = 0.55A$$

Ans.

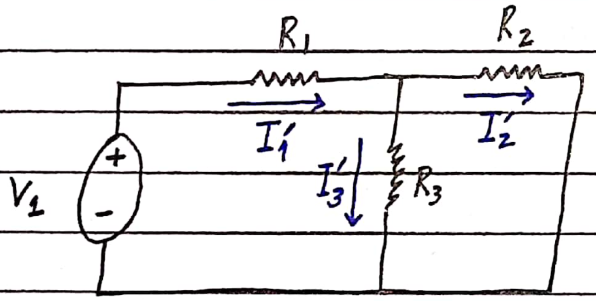
Superimposition Theorem

This theorem finds use in solving a network where two or more sources are present and not connected in series or parallel.

In a linear bilateral network containing several energy sources, the overall current in any branch is equal to the algebraic sum of current produced by each sources acting alone while the other sources are inactive.



Case 1: When $V_1 \neq 0$, $V_2 = 0$ (inactive)



Consider a complicated network having two voltage sources V_1 and V_2. Let us calculate the current in each branch using superposition

According to superposition theorem, select a single source in the given network that means $V_1 \neq 0$ and $V_2 = 0$ (short circuited).

$$R_{eq} = \frac{R_2 \cdot R_3}{R_2 + R_3} + R_1$$

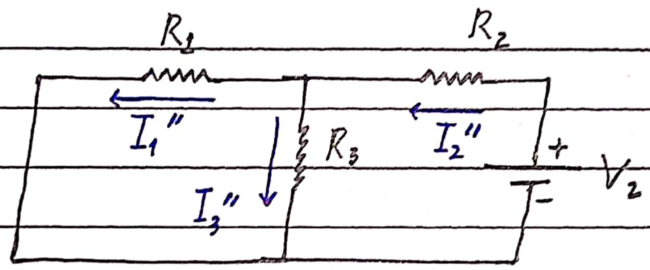
$$\therefore \quad I_1' = \frac{V_1}{\left(\frac{R_2 \cdot R_3}{R_2 + R_3} + R_1\right)}$$

and,
using current divider rule,

$$I_2' = \frac{I_1' R_3}{R_2 + R_3}.$$

$$I_3' = \frac{I_1' R_2}{R_2 + R_3}.$$

Case 2: When $V_2 \neq 0$, $V_1 = 0$ (inactive)



$$R_{net} = (R_1 \parallel R_3) + R_2$$
$$= \left(\frac{R_1 R_3}{R_1 + R_3} + R_2\right)$$

$$I_2'' = \frac{V_2}{R_{net}}$$

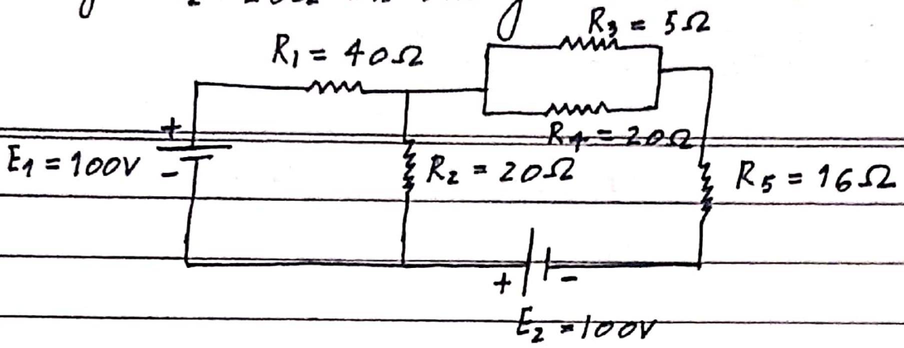
and $I_1'' = \dfrac{R_3}{R_1 + R_3} \times I_2''$

$$I_3'' = \frac{R_1}{R_1 + R_3} \times I_2''$$

or $\quad I_3'' = I_2'' - I_1''$

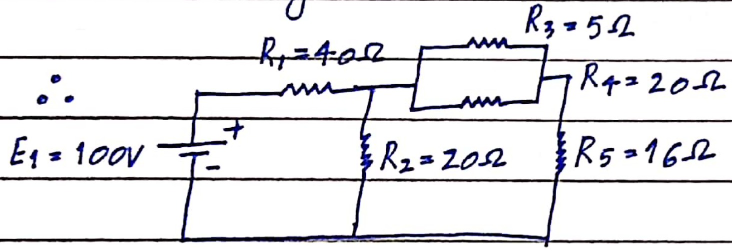
$\therefore$
$$I_1 = I_1' - I_1''$$
$$I_2 = I_2' - I_2''$$
$$I_3 = I_3' - I_3''.$$

Example: Utilising superposition theorem, find the current through $R_2 = 20\Omega$ in the given network.



Considering E_1 as active source and rest as inactive

$\therefore$



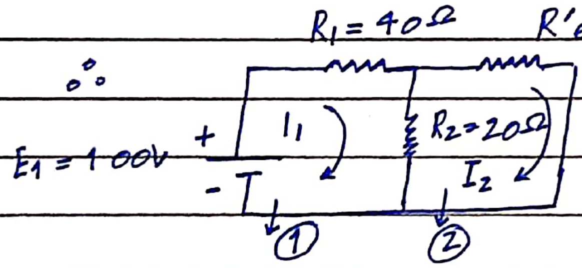
$\therefore$ R_3 and R_4 are in parallel combination

$$\Rightarrow R_{eq} = \frac{\cancel{5}\times\cancel{20}\,4}{\cancel{25}\,\cancel{5}} = 4\,\Omega$$

and R_{eq} and R_5 are in series

$$\Rightarrow R'_{eq} = 4 + 16 = 20\Omega$$

$\therefore$



$\therefore$ Applying KVL in mesh 1

$$100 - 40I_1 - 20(I_1 - I_2) = 0$$
$$-6\cancel{0}I_1 + 2\cancel{0}I_2 = -1\cancel{0}0$$
$$3I_1 - I_2 = 5 \quad \text{— (1)}$$

Applying KVL in mesh 2

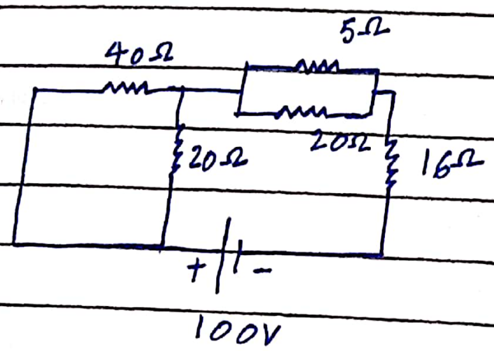
$$-20I_2 - 20(I_2 - I_1) = 0$$
$$-40I_2 + 20I_1 = 0$$
$$I_1 = 2I_2 \quad \text{— (2)}$$

$\therefore$ $6I_2 - I_2 = 5 \Rightarrow \boxed{I_2 = 1A}$ and $\boxed{I_1 = 2A}$

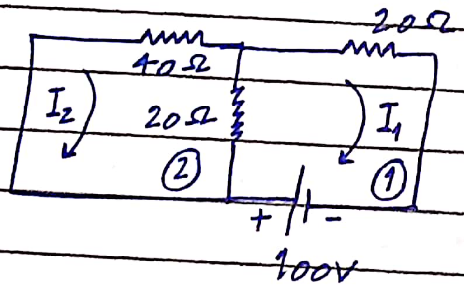
Hence,

$$I_{R_2=20\Omega} = I_1 - I_2 = 2-1 = 1\,A.$$

Now, considering E_2 as active source and others as inactive sources.



This network can be simplified into;



∴ Applying KVL in mesh 1

$$100 - 20(I_1 - I_2) - 20I_1 = 0$$
$$-40I_1 + 20I_2 = -100$$
$$2I_1 - I_2 = 5 \quad \text{--- (1)}$$

Applying KVL in mesh 2

$$-40I_2 - 20(I_2 - I_1) = 0$$
$$-60I_2 + 20I_1 = 0$$
$$I_1 = 3I_2 \quad \text{--- (2)}$$

∴ $6I_2 - I_2 = 5$

$\boxed{I_2 = 1A}$, $\boxed{I_1 = 3A}$

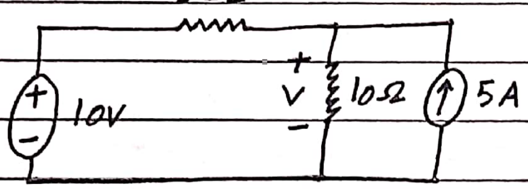
∴ $I'_{R_2=20\Omega} = I_1 - I_2 = 3-1 = 2A$

Current through $R_2 = 20\Omega$, when both sources are active

$$(I_{R_2=20\Omega})_{\substack{\text{Both}\\ \text{source}\\ \text{active}}} = (I_{R_2=20\Omega})_{\substack{E_1=\text{active}\\ E_2=0}} + (I'_{R_2=20\Omega})_{E_2=\text{active}}$$

$$I_{R_2=20\Omega} = 1+2 = 3\text{ A} \quad \underline{\underline{\text{Ans.}}}$$

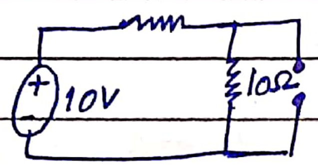
Q. Find the voltage V by using superimposition theorem in the given ~~nature~~ network.



★ دھیان دے!!

Inactive state:
Voltage source is replaced by a short wire i.e short circuited while current source is replaced by an open wire i.e it is opened.

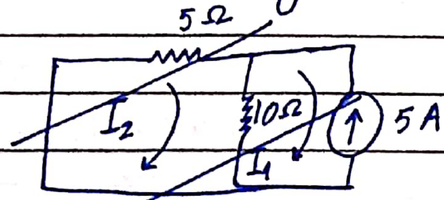
First considering 10V as an active source.



$\therefore$ $I = \dfrac{10}{15} = \dfrac{2}{3}$

$\therefore$ $V'_{10\Omega} = 10 \times \dfrac{2}{3} = 6.66\text{ V}$

Now, considering 5A as an active source;



~~$I_1 = -5A$~~

$\therefore$ Applying KVL for ~~mesh (1)~~

~~$-10(I_1 - I_2) = 0$~~

~~$10I_1 = 10I_2$~~

~~$I_1 = I_2$~~

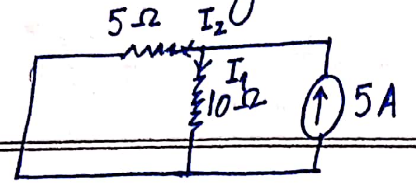
~~$-5I_2 - 10(I_2 - I_1) = 0$~~

~~$-15I_2 + 10I_1 = 0$~~

~~$\therefore \frac{3}{2}I_2 = I_1$~~

Now, considering 5A as an active source

∴



∴ Using Current Divider rule

$$I_1 = \frac{\cancel{5}\times 5}{\cancel{15}\,3} = \frac{5}{3}\ A$$

$$\Rightarrow V''_{10\Omega} = \frac{10\times 5}{3} = \frac{50}{3}\ V = 16.66V$$

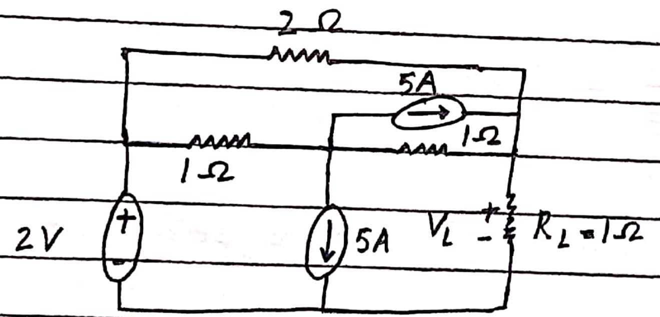
∴ Voltage across 10Ω when both sources are active ;

$$\Rightarrow V_{10\Omega} = V'_{10\Omega} + V''_{10\Omega}$$

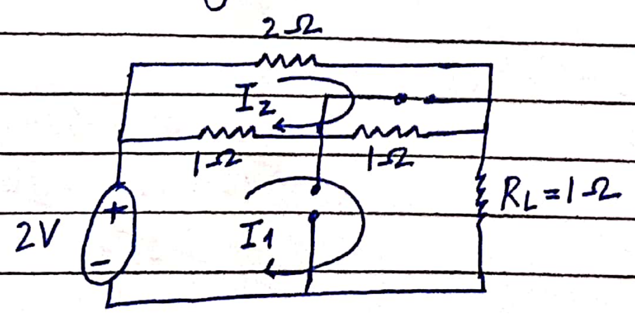
$$= 6.66 + 16.66$$

$$V_{10\Omega} = 23.32V \quad \text{Ans.}$$

Q. Find the voltage V_L in the given network using superposition theorem.



Considering 2V as an active source ;



Applying KVL in mesh 1

$-2(I_1-I_2)-I_1+2=0$

$-3I_1+2I_2=-2$ —— (1)

Applying KVL in mesh 2

$-2(I_2-I_1)-2I_2=0$

$-4I_2+2I_1=0$

$4I_2=2I_1$

$\boxed{I_1=2I_2}$ —— (2)

Putting (2) in (1)

$-6I_2+2I_2=-2$

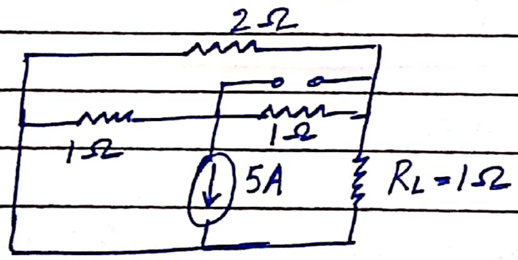
$-4I_2=-2$

$\boxed{I_2=0.5A}$

$\boxed{I_1=1A}$

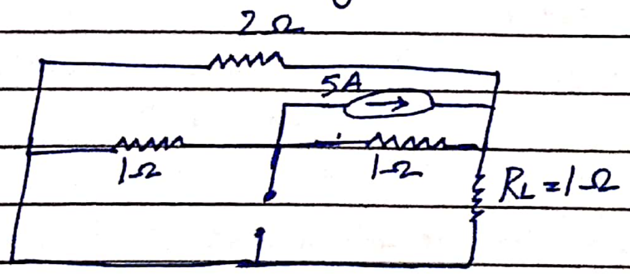
$\therefore \boxed{V_L'=1V}$

Now, considering '5A' as an active source



$\therefore$ $V_L''=5\times1=5V$

Now, considering '5A' as an active source



$\therefore$ $V_L'''=5\times1=5V$

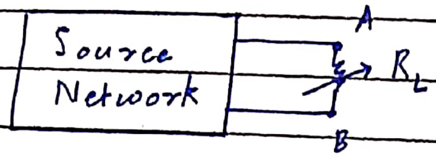
Hence $V_L=V_L'+V_L''+V_L'''$

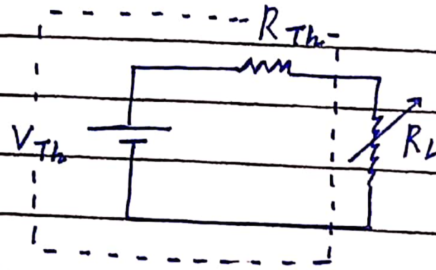
$V_L=1+5+5=11$ Ans.

Maximum Power Transfer Theorem

Maximum Power Transfer Theorem deals with power from a source to load. This theorem is used to find the value of load resistance for which there would be maximum amount of power from source to load.

A load resistance, being connected to a DC network receives maximum power when the load resistance is equal to the internal resistance of the source network.





$R_L = R_{Th}$
Maximum power delivered.

$$P = I_L^2 \cdot R_L$$

$$P = \frac{V_{th}^2}{(R_{th}+R_L)^2} \cdot R_L$$

$$P = \frac{V_{th}^2}{(2R_L)^2} \cdot R_L$$

$$\boxed{P_{max} = \frac{V_{th}^2}{4R_L}}$$

This theorem states that maximum power from a complex network is obtained when the load resistance R_L is equivalent to Thevenin's equivalent resistance R_{th} as seen from the load terminals.

Consider a variable resistance R_L is connected to DC source network having V_{Th} (Thevenin's equivalent voltage) and Thevenin's equivalent resistance R_{Th} of the source network. Therefore, we know that $\frac{V_{Th}}{R_{Th}+R_L} = I_L$ and the power delivered to load resistor is

$$P = I_L^2 \times R_L$$

$$P = \left(\frac{V_{Th}}{R_{Th}+R_L}\right)^2 \times R_L$$

Power P can be maximized by varying R_L when maximum power can be delivered to the load. When

$$P_L(max) = \frac{dP}{dR_L} = 0$$

$$\therefore \quad \frac{d}{dR_L}\left(\left(\frac{V_{th}}{R_{th}+R_L}\right)^2 \times R_L\right) = 0$$

$$R_L \cdot V_{th}^2 \left(\frac{d\,(R_{th}+R_L)^{-2}}{dR_L}\right) + \frac{V_{th}^2}{(R_{th}+R_L)^2} = 0$$

$$\frac{R_L \times V_{th}^2 \times (-2)}{(R_{th}+R_L)^3} + \frac{V_{th}^2}{(R_{th}+R_L)^2} = 0$$

$$\therefore \quad -2 \times R_L \times \cancel{V_{th}^2} = -\cancel{V_{th}^2} \times R_{th} - \cancel{V_{th}^2} \cdot R_L$$

$$\Rightarrow \boxed{R_L = R_{Th}}$$

Hence, $R_L = R_{Th}$, the system being perfectly matched for load and source. The power transfer becomes maximum and this amount of power is

$$P = \frac{V_{th}^2}{(R_L+R_{Th})^2} \times R_L = \frac{V_{th}^2}{4R_L^2} \times R_L = \frac{V_{th}^2}{4R_L}.$$

→ Efficiency (η): $\dfrac{\text{Output Power}}{\text{Input Power}}$

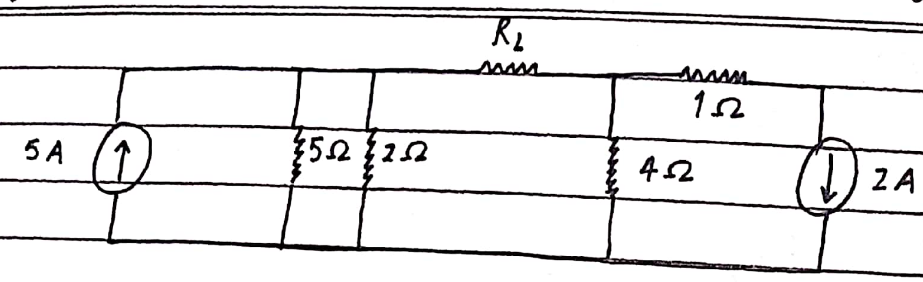
$$= \frac{I_L^2 \cdot R_L}{I_L^2\,(R_{Th}+R_L)} = \frac{R_L}{R_{th}+R_L}$$

Therefore maximum efficiency:

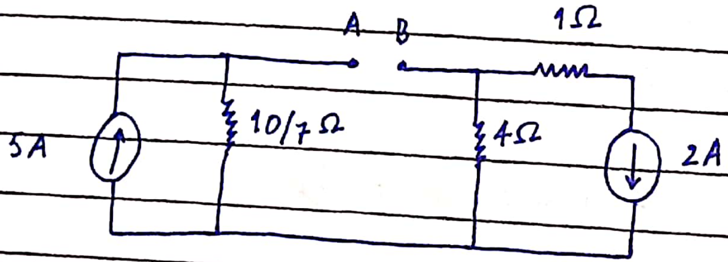
$$P = \frac{R_{Th}}{2R_{Th}} = 0.5 = 50\%$$

NOTE: Efficiency in maximum power transfer is only 50% as one half of the total power generated in internal resistance of the source network.

Q. Find the value of R_L such that maximum power transfer takes place from the current sources to the load resistor R_L and also obtain the amount of maximum power transferred in the given network.



Removing load resistance and calculating V_{Th}.

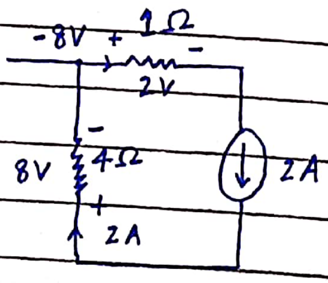


Replacing 5Ω and 2Ω with their eq. resistance.

$\therefore$ $R_{eq} = (5 \| 2) = 10/7\,\Omega$

$$\therefore \quad V_A = 5\left(\frac{10}{7}\right) = \frac{50}{7} = 7.14\,V$$

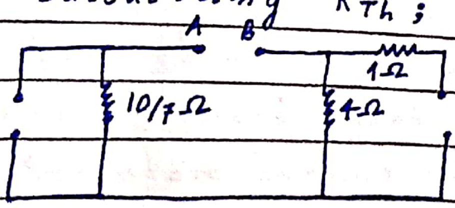
and



$\therefore$ $V_B = -8\,V$

$\therefore$ $V_{AB} = (7.14) - (-8) = 15.14\,V = V_{Th}$

Now, calculating R_{Th};



$\therefore$ $R_{Th} = \frac{10}{7} + 4 = 5.43\,\Omega$

$\therefore$ To transfer maximum power;

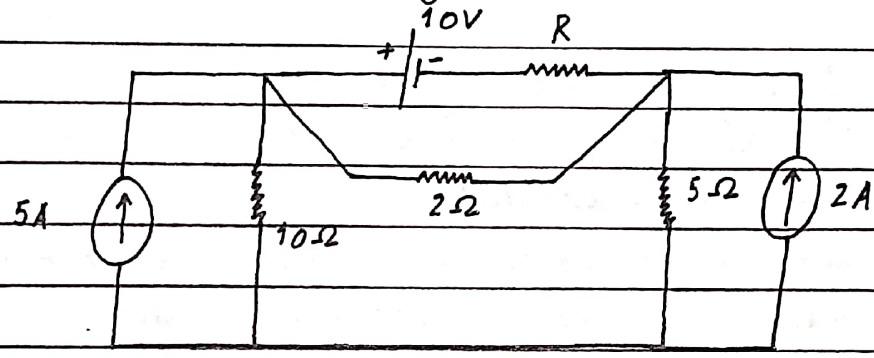
$R_L = R_{th} = 5.43\,\Omega$

According to Maximum Power Transfer Theorem;

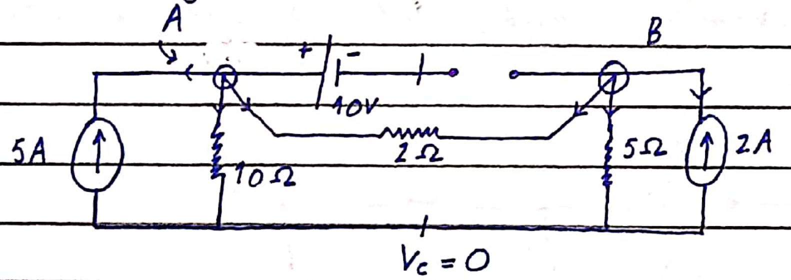
$$P_{Max} = \frac{V_{th}^2}{4R_L} = \frac{(15.14)^2}{4 \times 5.43} = 10.553\ \text{W}$$

Ans.

Q. Obtain maximum amount of power in the resistor R from the source using maximum power transfer theorem



Removing 'R' and calculating V_{th}



Applying nodal analysis at 'A'

$$-5 + \frac{V_A}{10} + \frac{V_A - V_B}{2} = 0$$

$$6V_A - 5V_B = 50 \quad (1)$$

Applying Nodal Analysis at B

$$\therefore \frac{V_B - 10}{2} + \frac{V_B}{5} - 2 = 0$$

$$7V_B - 5V_A = 20 \quad (2)$$

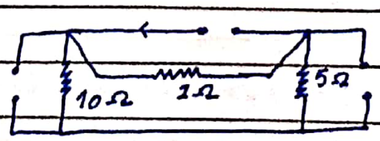
On solving (1) and (2), we get

$V_A = 26.46\,V$

$V_B = 21.76\,V$

$\therefore V_{AB} = 26.46 - 21.76 = 5.3\,V = V_{th}$

Now, calculating R_{Th};



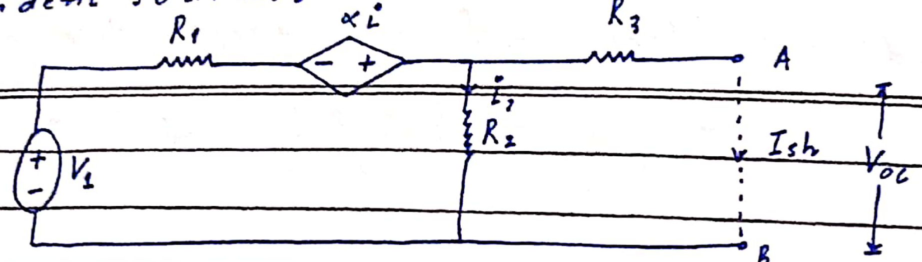
$R_{Th} = (10+5) \| 2 = 1.76\,\Omega$

$$\therefore P_{Max} = \frac{V_{th}^2}{4R_L} = \frac{5.3 \times 5.3}{4 \times 1.76 \times 1.76} = 3.990 \approx 4\,W$$

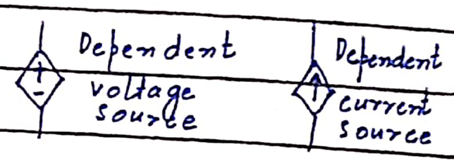
Ans.

Dependent Source

When circuit contains both and dependent and independent sources.



$$R_{th} = \frac{V_{oc}}{I_{sh}}.$$



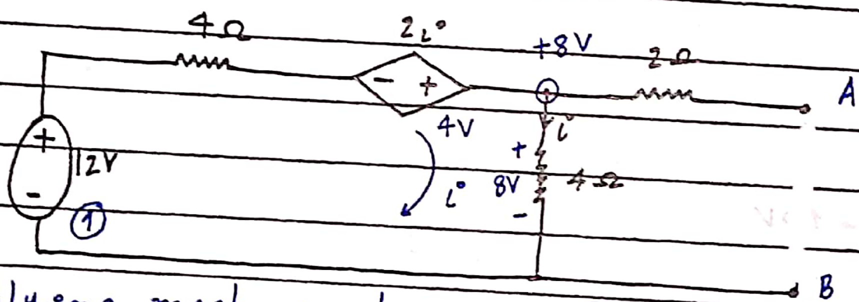
→ Procedure for finding V_{Th} and R_{Th}.

Step 1: Calculate the open circuited voltage V_{oc} as usual with all sources as activated.

Step 2: A short circuit is applied across the open terminal and find the value of short circuited current.

Step 3: Find Thevenin's equivalent resistance.

Q. Find the Thevenin's equivalent network for the given network.



Applying mesh analysis in 1

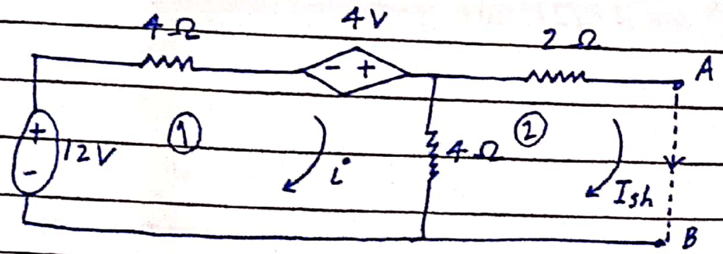
$\therefore$ $12 - 4i + 2i - 4i = 0$

$12 - 6i = 0$

$i = 2$

$\therefore V_{oc} = 8V$

$\therefore$



Short-circuiting terminal AB and calculating Ish by using mesh analysis

$\therefore$ Applying mesh analysis in (1)

$$12 - 4i + 4 - 4(i - i_{sh}) = 0$$

$$16 - 8i + 4i_{sh} = 0 \quad \text{—(1)}$$

Applying mesh analysis in (2)

$$4(i_{sh} - i) + 2i_{sh} = 0$$

$$6i_{sh} = 4i$$

$$3i_{sh} = 2i$$

Putting this in (1)

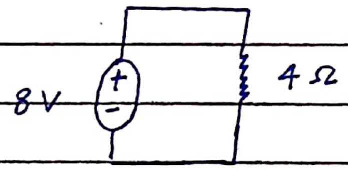
$$16 - 12i_{sh} + 4i_{sh} = 0$$

$$16 - 8i_{sh} = 0$$

$$\boxed{i_{sh} = 2\ A}$$

$\therefore$ $R_{th} = \dfrac{V_{oc}}{i_{sh}} = \dfrac{8}{2} = 4\Omega$

$\therefore$ Thevenin's equivalent network is



Ans.

Semiconductor Diode

When a p type semiconductor having acceptor impurities, is joined to a n-type semiconductor having donor impurities, the thin region in which the carrier changes from p-type semiconductor to n-type semiconductor is known as a pn junction.

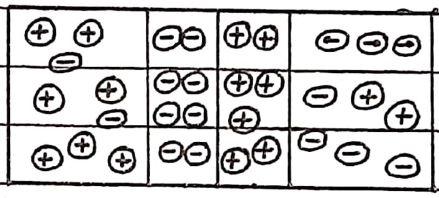
A pn junction forms a very useful device and is called a semiconductor diode or a pn junction diode. All the semiconductor devices contains atleast one pn junction. Therefore it is very important to understand the behaviour of pn junction, when connected in an electronic circuit.

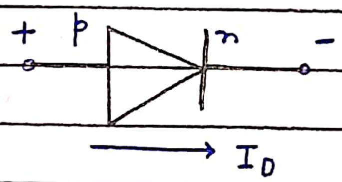
1N|4004

1 → 1 PN Junction

N → 4 Peak inverse voltage (mV)

4004 → Max current (mA)





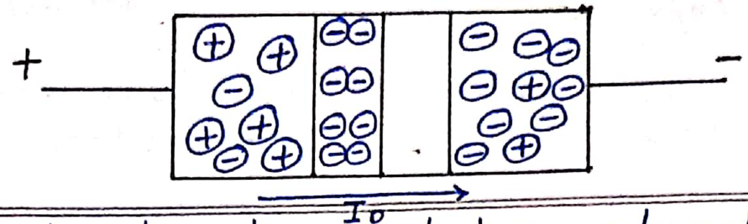
pn - junction

→ Formation of depletion layer is pn junction diode.

→ Biasing of PN-Junction Diode

When a pn junction diode is connected across an external voltage source, the diode is said to be under biasing condition. The external voltage source across the pn-junction diode can be applied in two way.

1. Forward Biasing
2. Reverse Biasing

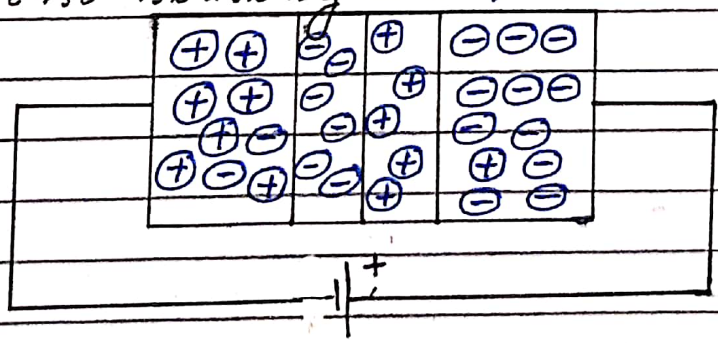


When positive terminal of an external voltage source, p-type semiconductor and negative terminal is connected to n-type semiconductor of pn junction diode is said to be in formed biasing.

The holes are repelled by positive terminal and electron is repelled by negative terminal of the voltage source and move towards the pn junction.

Because, they acquire energy from voltage source, same of the holes and electrons enters in the depletion layer and combine with each other width of depletion layer and barrier potential reduces with forward bias. As, a result of this more majority carrier defuse across the pn junction. Therefore it causes a large current to flow to the pn junction. This current is called forward current I_D, resulting the forward current to rise.

Reverse Biasing $\xrightarrow{I_s}$



When the positive terminal of an external voltage source is connected to n-type semiconductor and negative terminal is connected to p-type semiconductor, the pn junction diode is said to be in reverse biasing.

The holes in p-type semiconductor are attracted towards the negative terminal of voltage source and the electron in n-type semiconductor are attracted to posibive terminal of the voltage source.

Therefore, the majority carriers drawn away from the junction. The increased barrier potential makes it very difficult for the majority carriers to defuse across the pn junction, therefore no current flows due to majority carriers in a reversed biased pn junction diode.

But, the barrier potential helps minority carriers in crossing the pn junction, therefore a small current does flow through the reverse bias pn junction diode.
This current is known as reverse saturation current Is, at this action occurs is known as Breakdown voltage.

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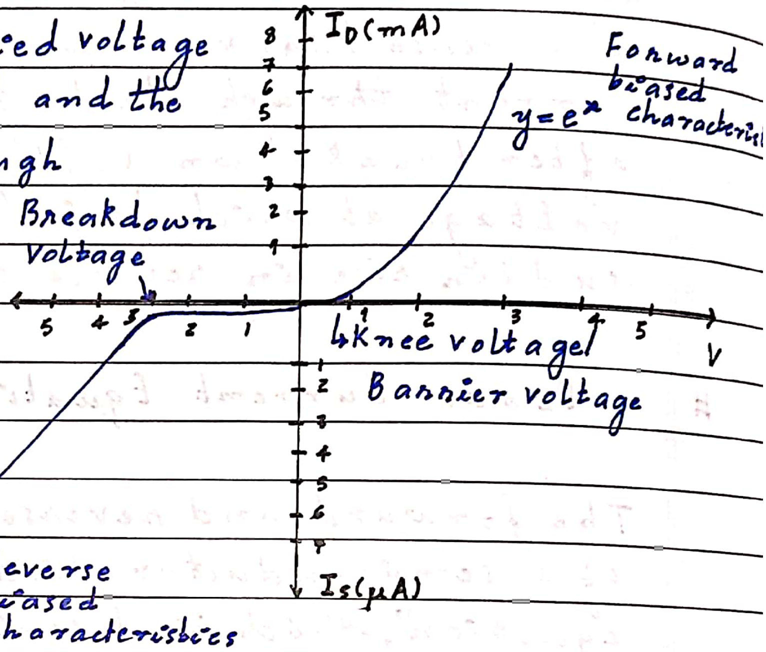
→ VI characteristics of PN Junction Diode

It is the graph between applied voltage across the diode terminal and the current that flows through the diode.

It may be noted that the complete graph can be divided into two parts.



First is Forward biased characteristics and Reversed biased characteristics.

→ Forward Biased Characteristics

In Forward bias, the pn junction barrier potential is reduced after knee voltage. (The minimum voltage applied to the pn junction diode for which it starts current conduction is known as 'knee voltage').

Due to this voltage and forward bias voltage, current increases with the increase in forward bias voltage.

The forward current increases as the voltage exceeds, the pn junction diode behaves like a conductor.

Therefore, the current rises exponentially.

→ Reverse Biased Characteristics

With reverse biased characteristics, the pn junction potential barrier is increased. Therefore, no current flows through the pn-junction diode but actually

Date __/__/____ Page No.: ____

a very small current flows through the diode due to movement of minority charge carriers. If reverse bias voltage is made too high, the current through the pn junction diode increases after breakdown voltage. (It is the reverse voltage at which pn junction breakdown with sudden rise in reverse current).

Diode Current Equation. (Schockley's Diode equation)

The forward and reverse biased characteristics of a semiconductor diode is called "diode current equation", which is described as the current equation for forward and reverse ~~d~~ biased diode is given by the relation :

$$I_D = I_s \left(e^{V/\eta V_T} - 1 \right)$$

where,

$V \rightarrow$ diode terminal voltage
It is positive for forward bias and negative for reverse bias.

$\eta \longrightarrow$ material or emperical constant
$\begin{Bmatrix} \eta = 2 \text{ for Silicon} \\ \eta = 1 \text{ for Germenium} \end{Bmatrix}$

$V_T \rightarrow$ Volt equivalent temperature / Thermal voltage

$$V_T = \frac{KT}{q},$$

$K \rightarrow$ Boltzmann constant $= 1.38 \times 10^{-23}$ J/K
$T \rightarrow$ Temperature in K
$q \rightarrow$ electron charge $= 1.6 \times 10^{-19}$ C

Calculating V_T for room temperature

$\therefore T = 298K$

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$$\Rightarrow V_T = \frac{1.38\times10^{-23}\times 298}{1.6\times10^{-19}} = 0.025V = 25mV$$

$I_S \rightarrow$ Reverse Saturation Current

$I_D \rightarrow$ Diode current.

→ When V is positive and $V >>> V_T$, then the term $e^{V/\eta V_T} >>> 1$.

Therefore, the diode equation becomes;

$$I_D = I_S \times e^{V/\eta V_T}$$

→ If V is negative and $V <<< V_T$, then the term $e^{V/\eta V_T} <<< 1$.

Therefore, the diode current under reverse bias is equal to the reverse saturation current I_S.

$$\therefore I_D = -I_S.$$

Q. Determine Germenium pn-junction diode current for the forward biased voltage of 0.22 V at room temperature with I_S of 1mA.

$$\therefore I_D = I_S \times e^{V/\eta V_T}$$

$$= (1 mA)\times e^{\frac{0.22}{1\times0.025}}$$

$$I_D = (1mA)\times(6634) = 6634 mA$$

$$I_D = 6.634 A$$ ans.

"Patience is a key element of success."—Bill Gates

Q. Derive the equation for the ratio of diode current resulting from two different forward voltage drop if $V >>> V_T$.

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$$\therefore \quad I_{D_1} = I_s \left(e^{V_1/\eta V_T} \right)$$

$$I_{D_2} = I_s \left(e^{V_2/\eta V_T} \right)$$

Hence :

$$\frac{I_{D_2}}{I_{D_1}} = e^{(V_2 - V_1)/\eta V_T} = e^{\Delta V/\eta V_T}.$$

Q Find the increase in forward voltage if the current of Si diode with $V_T = 26mV$ is doubled.

$$\therefore \quad \frac{I_{D_2}}{I_{D_1}} = e^{\Delta V/\eta V_T}$$

$$\therefore \quad \ln\left(\frac{I_{D_2}}{I_{D_1}}\right) = \frac{\Delta V}{\eta V_T}$$

$$\Delta V = \eta \times V_T \times \ln\left(\frac{I_{D_2}}{I_{D_1}}\right)$$

$$\Delta V = 2 \times 0.026 \times \ln\left(\frac{2 I_{D_1}}{I_{D_1}}\right)$$

$$\Delta V = 2 \times 0.026 \times \ln(2)$$

$$\Delta V = 0.052 \times 0.693$$

$$\Delta V = 0.036V$$

$$\Delta V = 36mV$$ ans.

Q. The diode current is 0.6 mA where the applied voltage 400 mV and 20 mA when applied voltage 500 mV.

Date __/__/__ Page No.: ____

Determine emperical constant η at room temperature.

$$\frac{I_{D_2}}{I_{D_1}} = e^{\frac{\Delta V}{\eta V_T}}$$

$$\therefore \frac{\Delta V}{\eta \times V_T} = \ln\left(\frac{I_{D_2}}{I_{D_1}}\right)$$

$$\therefore \eta = \frac{\Delta V}{V_T \times \ln\left(\frac{I_{D_2}}{I_{D_1}}\right)}$$

$$\eta = \frac{0.5 - 0.4}{0.025 \times \ln\left(\frac{20}{0.6}\right)}$$

$$\eta = 1.140 \quad \text{Ans.}$$

Diode Resistance
The practical diode does not behave as a perfect conductor when it is forward biased, similarly it does not behave as a perfect insulator, when it is reversed bias due to diode resistance.

In other words when a practical diode is forward biased, it offers low resistance to the forward current and when the practical diode is reversed biased it offers very high resistance.

The diode resistance can be classified based on biasing conditions.
There are two types
(i) Forward Diode resistance.
(ii) Reverse Diode resistance.

→ Forward Diode Resistance
Under the forward biased condition, the opposition offered by the diode is known as forward diode resistance. It can be defined in two manners:

(a) Static Forward Diode Resistance
When the diode is used in DC circuit and supplied voltage is DC, the resistance is calculated at a particular point on the VI characteristics of forward bias and the resistance is called Static Forward Diode resistance.

$$R_{F(DC)} = \frac{V_D}{I_D}.$$

(b) Dynamic Forward Diode Resistance
The opposition offered by diode to the alternative current flows in forward bias condition is known as Dynamic Forward Resistance or AC resistance.

$$R_{F(DC)} = \frac{\Delta V_D}{\Delta I_D} = \frac{dV}{di}$$

(ii) Reverse Diode resistance

Under reverse bias condition, the opposition offered by the diode to the reverse saturation current is known as reverse diode resistance.
It can also be expressed in two manners;

(a) Static Reverse Diode Resistance

The resistance under DC conditions, it is the ratio of applied reverse voltage to the reverse saturation current.

$$R_{R(DC)} = \frac{V_R}{I_S}$$

(b) Dynamic Reverse Diode Resistance

The resistance under AC condition, it is the ratio of change in reverse voltage to the corresponding change in reverse saturation current.

$$R_{R(AC)} = \frac{\Delta V_R}{\Delta I_S} = \frac{dV_R}{dI_S}$$

→ Expression for Dynamic Diode Resistance

$$R_{(AC)} = \frac{1}{\left(\frac{dI}{dV}\right)} \quad \text{——— (1)}$$

From Diode current equation:

$$I_D = I_S\left(e^{V/nV_T} - 1\right) \quad \text{——— (2)}$$

Differentiating eq ② w.r.t V

$$\frac{dI_D}{dV} = \frac{I_s \cdot e^{V/\eta V_T}}{\eta V_T}$$

$\therefore$ Putting in eq ①

$$\therefore R_{r(AC)} = \frac{\eta V_T}{I_s \cdot e^{V/\eta V_T}} = \frac{\eta V_T}{I_D + I_s}$$

From the above equation determining the value of dynamic diode resistance under forward and reverse bias conditions.

Q. Find the static and Dynamic resistance of Ge diode, if temperature is 27°C and saturation current is 1μA for applied forward bias voltage of 0.2V.

$$\therefore V_T = \frac{KT}{q} = \frac{1.38 \times 10^{-23} \times 300}{1.6 \times 10^{-19}} = 0.0258$$

$$\Rightarrow V_T = 0.026 \text{ V}$$

$$\therefore I_D = I_s (e^{V/\eta V_T})$$
$$= 1 \times 10^{-6} \times e^{0.2/0.026}$$
$$I_D = 2.186 \times 10^{-3} \text{ A}$$

$$\therefore R_{F(DC)} = \frac{V_D}{I_D} = \frac{0.2}{2.186 \times 10^{-3}} = 91.491 \,\Omega$$

$$R_{F(AC)} = \frac{\eta V_T}{I_s + I_D} = \frac{0.026}{2.186 \times 10^{-3} + 10^{-6}} = 11.888 \,\Omega$$

Ans.

Q. Show that the pn junction resistance for diode is given by the equation

$r_j = \frac{26}{I_D}$ at 27°C where I_D is the forward diode current in mA.

$$\therefore \quad V_T = \frac{KT}{q} = \frac{1.38\times10^{-23}\times300}{1.6\times10^{-19}} = 26\,mV$$

$$\therefore \quad R = \frac{\eta V_T}{I_D + I_s}$$

Assuming $\eta = 1$ and

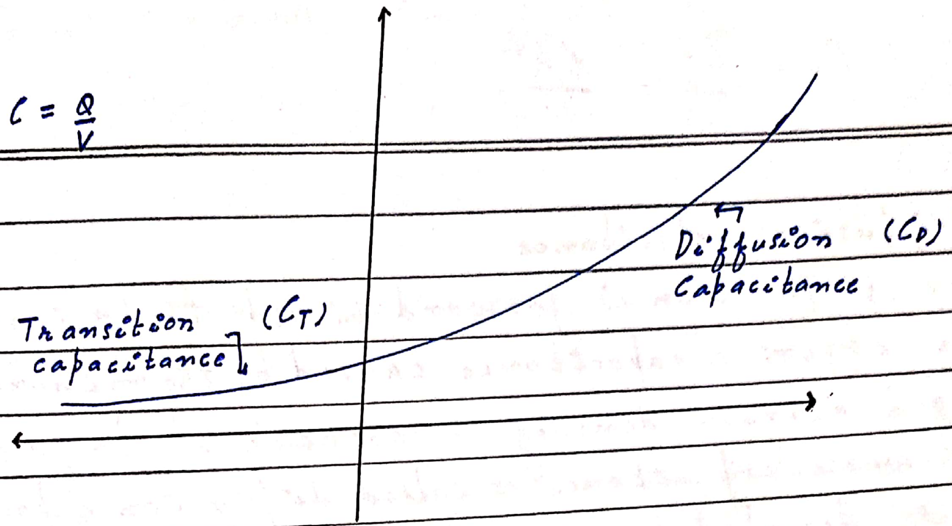
$I_D >>> I_s$

Hence,

$$R_j = \frac{V_T}{I_D} = \frac{26}{I_D(mA)}$$ Proved.

Diode Capacitance

$C = \frac{Q}{V}$



The depletion layer acts as a dielectric medium between p-type semiconductor and n-type semiconductor and as a parallel plates of capacitor. The capacitance formed in a junction area is called diode capacitance and it is given by the equation.

$$C = \frac{\epsilon A}{w}$$

where ϵ is the permitivitty of the dielectric medium
A is the area of the junction.
w is the width of the depletion layer

The pn junction diode grows in capacitance under biased conditions and it is of two types

1. Transition Capacitance
2. Diffusion Capacitance

Transition Capacitance

When the pn junction diode is reversed biased, than the majority carriers move away from the junction and covered more immobile charges. Therefore, the thickness of depletion layer increases with the increase of reverse bias voltage.

This depletion layer along with concentration of immobile charges maybe considered to a capacitor and whose capacitance is known as transition capacitance (C_T).

$$C_T = \frac{dq}{dV} = \frac{\epsilon A}{w}.$$

2. Diffusion Capacitance

When pn junction is forward biased, the pn junction diode offers a capacitance caused by the minority charged carrier density stored near the pn junction and whose capacitance is called diffusion capacitance. It is denoted by C_D and also expressed by

$$C_D = \frac{\tau \cdot I}{\eta V_T}$$

We know that diffusion capacitance

$$C_D = \frac{dq}{dV}$$

and also, we know that

$$di = \frac{dq}{dt}$$

if $dt = \tau$, then $dq = \tau \cdot di$

Therefore

$$C_D = \frac{\tau \cdot dI}{dV}$$

Hence,

$$\frac{dI}{dV} = \frac{I + I_s}{\eta V_T}$$

But $I >>> I_s$

then $\frac{dI}{dV} = \frac{I}{\eta V_T}$

Now, the diffusion capacitance

$$C_D = \frac{\tau I}{\eta V_T}$$

The diffusion capacitance is proportional to the diode current.

Q. Find the transition capacitance of Ge Diode whose $A = 1mm^2$ and space charge thickness is 2×10^{-4} cm and permitivity of Germanium diode is $\varepsilon_{Ge} = \frac{16}{36\pi}\times10^{-11}$ F/cm

$$C = \frac{\frac{16}{36}\pi\times10^{-11}\times0.01}{2\times10^{-4}}$$

$$C = 7.073\times10^{-11}\,F$$

→ Diode Applications

The pn junction diode has an important characteristics that it conducts well in forward direction and poorly in reverse direction. The pn junction diode is very useful in many application such as

→ as a switch in logic circuit

→ as a rectifier

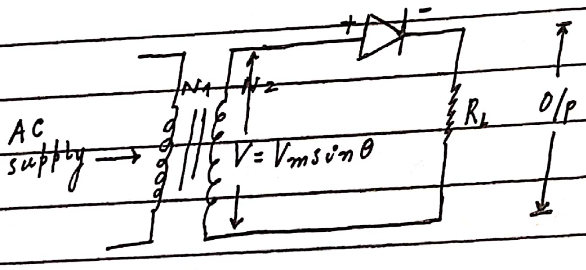
→ as a zener diode in voltage stabilizing circuit.

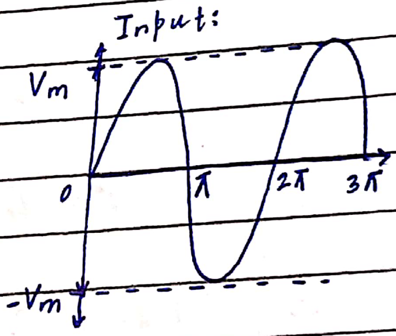
→ as a verector diode in radio and TV receivers.

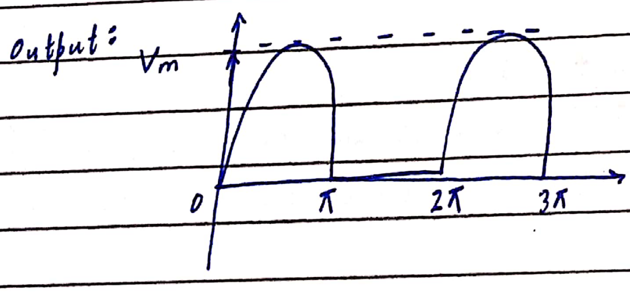
→ as a LED in communication circuits.

→ Diodes as rectifiers

1. Half-wave rectifier circuit







In half-wave rectifier circuit at the input only positive half cycle appears across the load, whereas the negative half-cycle is suppressed.

To determine the efficiency of half-wave rectifier the rectification efficiency is the percentage of total input power is converted into useful DC output power, the ratio of DC output power to the AC input power

$$\eta = \frac{P_{dc}}{P_{AC}} \times 100\%$$

Let the applied voltage across the secondary winding is $V = V_m \sin\theta$, and the forward resistance of diode R_f and load resistor R_L, then the current flows through the load resistor.

$$i = \frac{V_m \sin\theta}{R_f + R_L}$$

When $\sin\theta = 1$, the instantaneous current is maximum

$$i = \frac{V_m}{r_f + R_L}$$

$$i = i_0 \sin\theta$$

$$\therefore \quad P_{dc} = I^2_{dc} \times R_L$$

$$I_{av} = \frac{\int_0^{\pi} I_m \sin\theta \, d\theta}{\int_0^{2\pi} d\theta}$$

$$I_{av} = \frac{I_m}{\pi}$$

$$\Rightarrow \quad P_{dc} = \frac{I_m^2}{\pi^2} \times R_L$$

$\cos 2\theta = 1 - 2\sin^2\theta$

$\sin^2\theta = \dfrac{1-\cos 2\theta}{2}$

and $P_{AC} = I_{rms}^2 (r_f + R_L)$

$$\therefore \quad I_{rms} = \sqrt{\frac{\int_0^{\pi} i^2 \, d\theta}{\int_0^{2\pi} d\theta}} = \sqrt{\frac{i_m^2 \int_0^{\pi} \sin^2 d\theta}{2\pi}}$$

$\cos 2\theta$

$$I_{rms} = i_m \sqrt{\frac{\left[\frac{\theta}{2}\right]_0^{\pi} - \left[\frac{\sin 2\theta}{4}\right]_0^{\pi}}{2\pi}}$$

$$I_{rms} = \frac{i_m}{2}$$

$$\Rightarrow \quad P_{AC} = \frac{i_m^2}{4} (r_f + R_L)$$

Hence ; $\eta = \dfrac{P_{DC}}{P_{AC}}$

$$\eta = \frac{\cancel{I_m^2} \times R_L \times 4}{\pi^2 \times \cancel{I_m^2} (R_f + R_L)}$$

$$\eta = \frac{4}{\pi^2 \left(1 + \frac{r_f}{R_L}\right)}$$

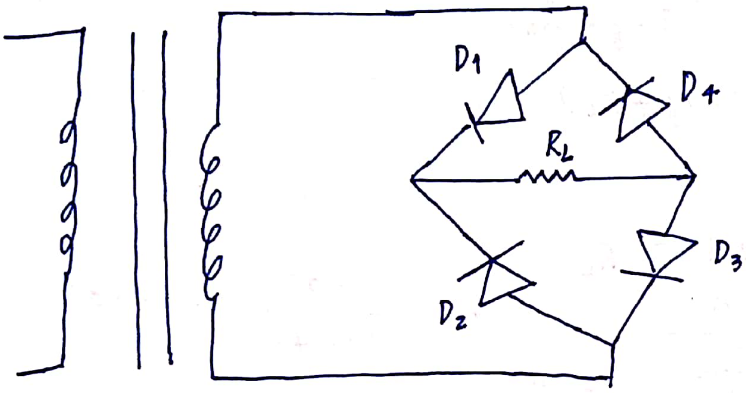
$$\eta = \frac{0.405}{1 + \frac{r_f}{R_L}}$$

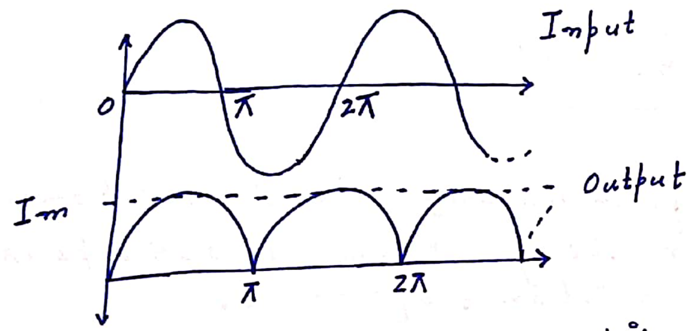
The effiency of rectifier will be maximum if r_f is neglected as compared to R_L.

Therefore, $\eta_{max} = 0.405 = 40.5\%$

Full wave-bridge rectifier

In this rectifier, circuit configuration an ordinary transformer is used and in place of center tap transformer and it contains four diodes connected from a bridge.



∴ 

$I = I_m \sin\theta$

Break this. → $2\int_0^{\pi} I_m \sin\theta \, d\theta + \int_{\pi}^{2\pi} \cancel{I_m \sin\theta \, d\theta}$ 0

∴ $P_{dc} = I_{dc}^2 \times R_L$

∴ $I_{dc} = I_{av} = \dfrac{\int_0^{2\pi} I_m \sin\theta}{\int_0^{2\pi} d\theta} = \dfrac{2I_m [\cos\theta]_0^{\pi}}{2\pi}$

$= \dfrac{4I_m}{2\pi} = \dfrac{2I_m}{\pi}$. Ans.

$\Rightarrow I_{dc} = I_{avg} = \dfrac{2I_m}{\pi}$

$I_{rms} = \sqrt{\dfrac{2\int_0^{\pi} I_m^2 \sin^2\theta}{2\pi}} = \dfrac{I_m}{\sqrt{2}}$

$\Rightarrow I_{AC} = I_{rms} = \dfrac{I_m}{\sqrt{2}}$

$\therefore \quad P_{DC} = I_{avg}^2 \cdot R_L$

$P_{DC} = \frac{4 I_m^2}{\pi^2} \cdot R_L$

$\therefore \quad P_{AC} = I_{rms}^2 \cdot (2r_f + R_L)$

$= \frac{I_m^2}{2} \cdot (2r_f + R_L)$

$$\Rightarrow \quad \eta = \frac{\frac{4 I_m^2}{\pi^2} \cdot R_L}{\frac{I_m^2}{2} \cdot 2r_f + R_L} = \frac{0.812}{2r_f + R_L} = 0.812 \quad (r_f = 0).$$

$$\boxed{\eta_{max} = 81.2\ \%}$$

Q. A half wave rectifier having a load resistance of 1000Ω, rectifier and alternating voltage of 325 V peak value and diode has forward resistance of 100Ω.

Calculate:

(i) Peak avg and rms value of current.

(ii) DC output power

(iii) AC input power

(iv) Rectification ~~power~~ efficiency.

$\therefore$ (i) $I_{Max} = \frac{V_{max}}{R_L + r_f} = \frac{325}{1100} = 0.2954\ A$

$I_{avg} = \frac{I_M}{\pi} = \frac{0.295}{\pi} = 94.04 \times 10^{-3}\ A$

$I_{rms} = \frac{I_M}{2} = \frac{0.2954}{2} = 147.70 \times 10^{-3}\ A$

$\therefore \quad P_{DC} = I_{avg}^2 \times R_L$

$= 8.84\ W$

$P_{AC} = I_{rms}^2 (r_f + R_L)$

$= 23.996\ W$

$$\therefore \eta = \frac{P_{DC}}{P_{AC}} \times 100 = \frac{8.84}{23.996} \times 100 = 36.81\%$$

Analyses of rectifier output

The output wave of full wave rectifier current flows in load resistor. In one direction only. But it varies in magnitude and pulsating in nature.

This pulsating output contains DC as well as AC components. It is clear that the rectifier outputs contains DC component and AC component, these undesirable AC component are called ripples.

The output of rectifier comprises of AC as well as DC component the % of AC component in rectified output is measured by ripple factor.

Ripple Factor (γ):

In a rectifier output, the ratio of RMS value of AC component to the DC component is known as ripple factor.

$$\gamma = \frac{\text{RMS value of AC component}}{\text{Value of DC component}}$$

$$\gamma = \frac{I_{ac}}{I_{dc}} \quad \text{and} \quad I_{ac}^2 = I_{rms}^2 - I_{dc}^2$$

$$\gamma = \frac{\sqrt{I_{rms}^2 - I^2_{dc}}}{I_{dc}} = \sqrt{\left(\frac{I_{rms}}{I_{dc}}\right)^2 - 1}$$

For half wave rectifier:

$$\gamma = \sqrt{\left(\frac{I_M \cdot \pi}{2 I_M}\right)^2 - 1} = 1.21$$

It mean the AC component exceeds the DC component when the output contains more ripple. Therefore, the half wave rectifier poorly converts AC into DC.

For full wave rectifier;

$$\gamma = \sqrt{\frac{(I_M/\sqrt{2})^2}{(2 I_M/\pi)^2} - 1} = \sqrt{(1.11)^2 - 1} = 0.48.$$

It means the AC component is very small compared to DC current, and output contains less ripple.

Therefore the full wave rectifier better converts AC into DC.

Q. In full wave centre-tap rectifier uses two diodes each having a forward resistance of 25Ω. The rms value of secondary winding voltage fade between centre tap. To each end of secondary winding is 48V and load resistance is 1 kΩ. Determine (i) DC output power.

(ii) AC input power

(iii) Rectification power

(iv) Ripple factor of rectifier.

$\therefore$ $V_{rms} = 48V$

$R_L = 1000\Omega$

$R_f = 25\Omega$

$$\therefore \quad I_M = \frac{V_M}{r_f + R_L} = \frac{48\sqrt{2}}{25+1000} = 66.33 \text{ mA}$$

$\Rightarrow$ (i) $P_{dc} = (I_{dc})^2 \times (R_L)$

$$= \left(\frac{2 I_M}{\pi}\right)^2 \times 1000 = 1.78 \text{ W}$$

(ii) $P_{AC} = (I_{rms})^2 \times (r_f + R_L)$

$$= \frac{(66.33 \times 10^{-3})^2}{(\sqrt{2})^2} \times (25 + 1000)$$

$P_{AC} = 2.254 \text{ W}.$

$$\eta = \frac{1.78}{2.254} \times 100 = 78.97\%$$

(iii) $r_f = 0.48.$

Q. In 240V, 50Hz AC supply is applied to a 5:1 step down transformer, which is used in full-wave bridge rectifier having a load resistance of 200Ω and each Si diode with 15Ω forward resistance. Calculate,

(i) DC voltage across the load.

(ii) Current flowing through the diode

(iii) Rectification efficiency and ripple factor.

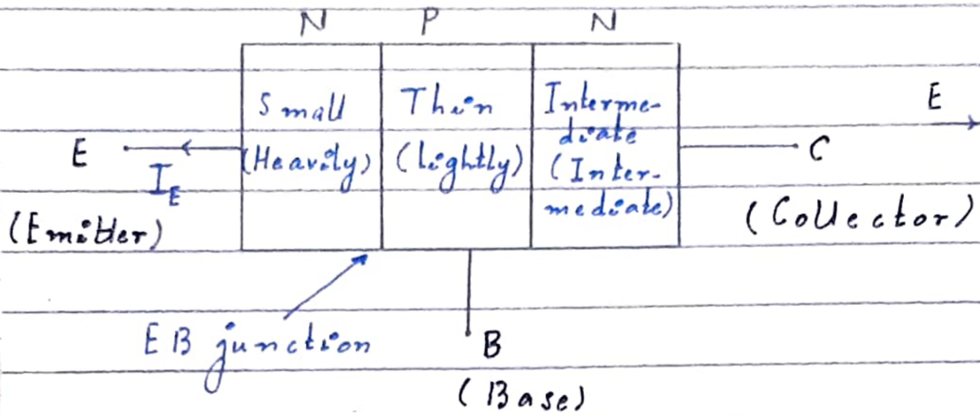
$$\therefore \quad \frac{V_1}{V_2} = \frac{N_1}{N_2}$$

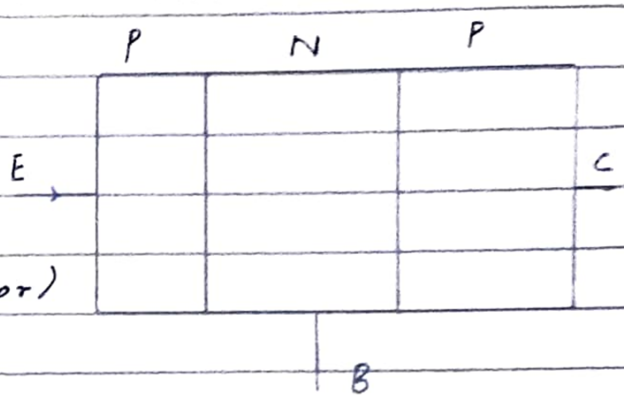
$$\therefore \quad V_2 = \frac{N_2 V_1}{N_1} = \frac{1 \times 240}{5} = 48\,V$$

$\therefore$ (i) $V_{rms} = 48\sqrt{2} = 6\cancel{7}.88\,V.$

Transistor

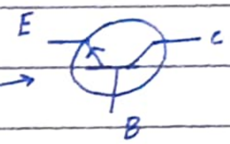
A transistor is a 3 layer semiconductor device consisting of 2 PN junction form by sandwitching either p type or n-type semiconductor between a pair of opposite type of semiconductor is called a transistor.



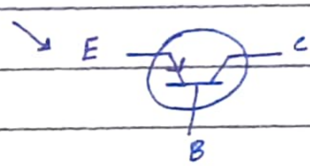


NPN Transistor

→ NPN Transistor



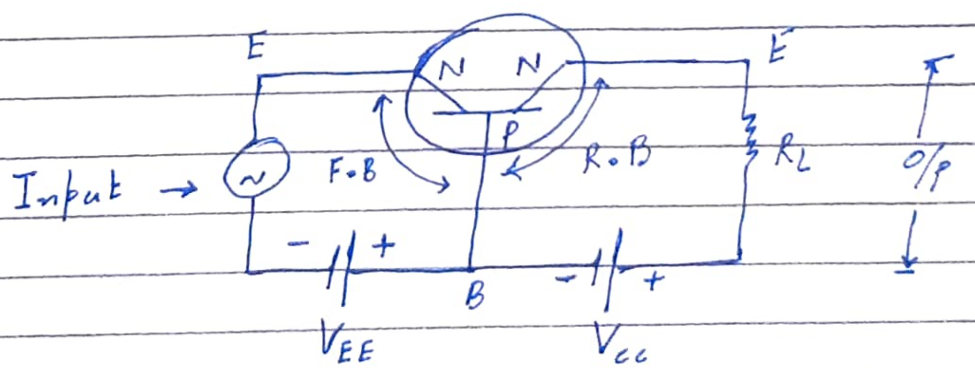
→ PNP Transistor



→ Transistor Biasing / Mode:

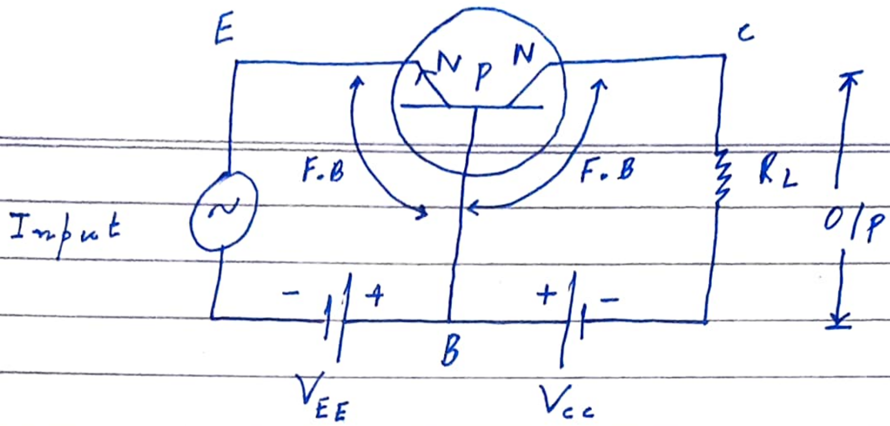
The methods of applying external voltage to a transistor is called transistor biasing. There are 3 different ways of biasing which are also known as mode of transistor operation.

1. Forward Active Mode.



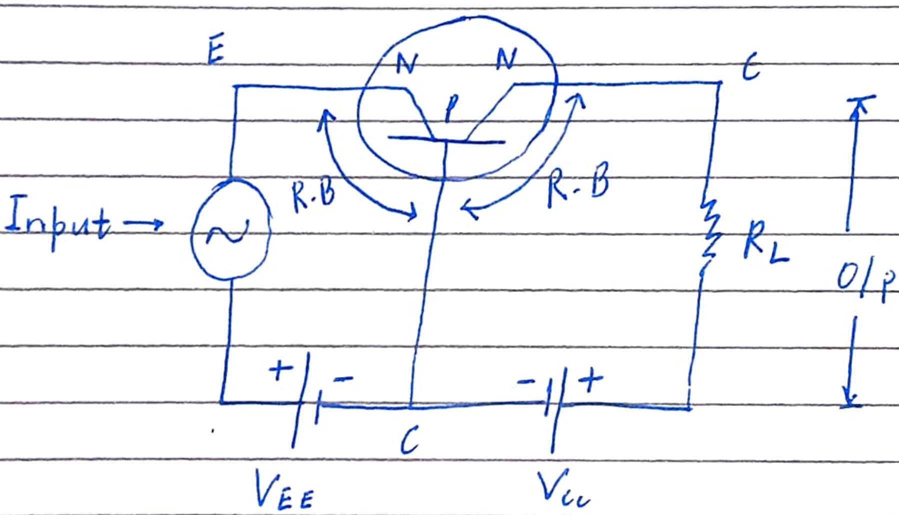
It works as an amplifier

2. Saturation Mode



It works as a~~n~~ closed switch.

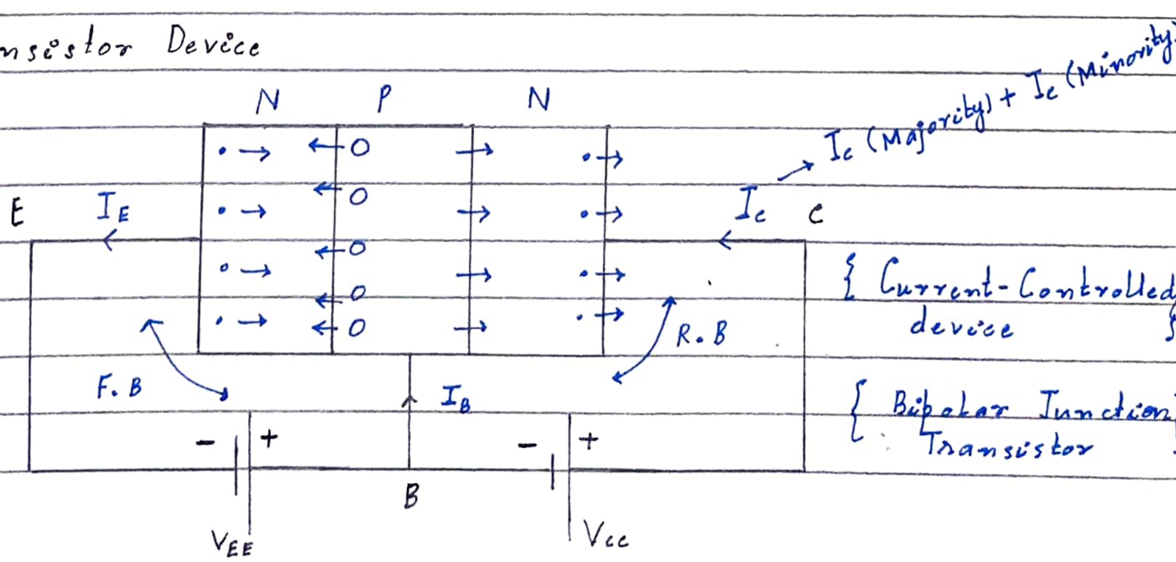
3. Cut-off Mode



It works as an open switch.

$$I_E = I_B + I_C$$

Transistor Device



{Current-Controlled device}

{Bipolar Junction Transistor}

The forward biased on emitter based junction causes the free $e^\ominus$ (majority carriers) in N-type emitter terminal to flow towards the base region due to this phenomena flows the emitter current I_E and when the free $e^\ominus$ enter into the p-type base region, they tend to combine with holes. Since the base region is lightly doped and very thin. Only few $e^\ominus$ combine with holes, due to this phenomena flows the base current I_B. Remaining $e^\ominus$ comes into the collector region and attracted by the collector, due to this phenomena flows the collector current I_C, therefore the emitter current is the sum of collector and base current.

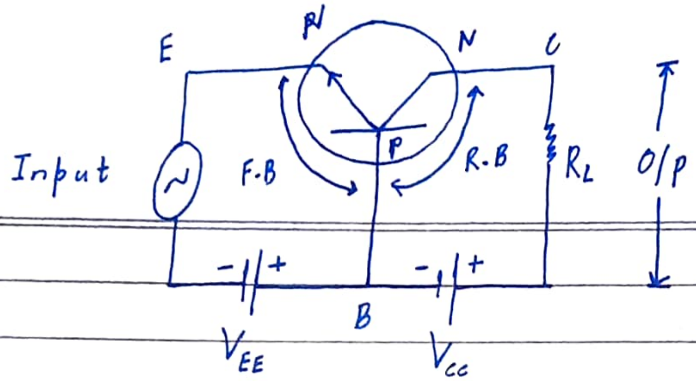
But, the collector current has two components, the majority and minority current carriers. The minority current carriers is called I_C (minority) or reverse leakage current, i.e

$$I_C = I_C\,(\text{majority}) + I_C\,(\text{minority})$$

NOTE :-
- → Transistor works as an amplifier.
- → It is a current controlled device
- → It is referred as bipolar junction transistor (BJT) because in this device current conduction takes place by the motion of charge carriers of both $e^\ominus$ and holes.
- → Both majority and minority carriers play important roles as a transistor.

Transistor Configuration

Transistor is configured based on common terminals, therefore transistor can be connected in an electronic circuit in three ways :



$$\text{Amplification Factor } (\alpha) = \frac{\text{Output current}}{\text{Input current}}$$

$$\alpha = \frac{I_c}{I_E} \qquad (\alpha = 0.9 \text{ to } 0.99)$$

In this configuration, the base terminal of this transistor of is made common to both input and output circuit however the emitter current is the sum of collector current and base current.

Now, the amplification factor or current gain is defined as the rate of output current to input current at constant constant collector base voltage and is denoted by α.

$$\alpha = \frac{\text{Output current}}{\text{Input current}}$$

$$\alpha = \frac{I_c}{I_E} \quad (\text{At constant } V_{CB})$$

$$\alpha_{AC} = \frac{\Delta I_c}{\Delta I_E} \quad (\text{At constant } V_{CB})$$

The value of current amplification factor α is also less than unity and it's value in commercial transistor range is 0.9 to 0.99.

$$\alpha = \frac{I_c}{I_E}$$

$$I_c(\text{majority}) = \alpha I_E$$

$$\therefore \quad I_C = \alpha I + I_{co}$$

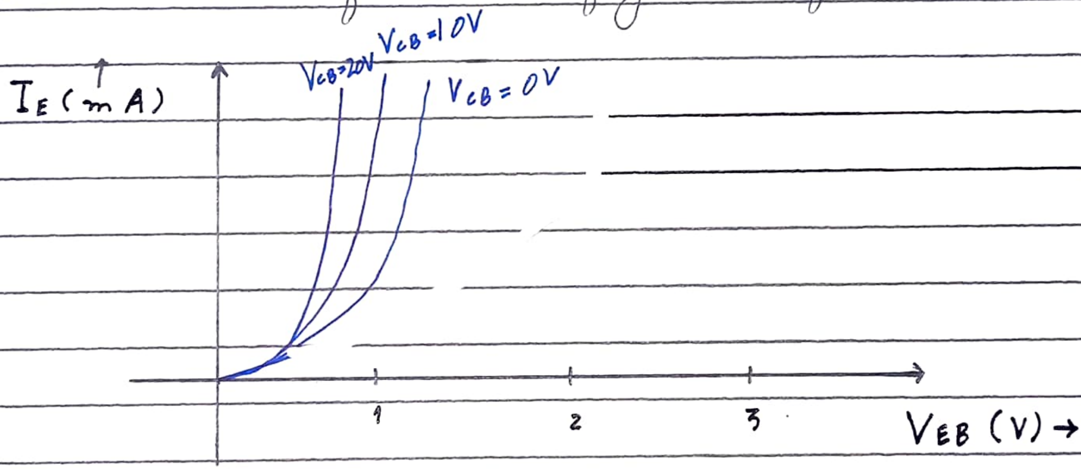
$$I_E = I_B + I$$

$$I_c = \alpha(I_B + I_c) + I_{co}$$

$$I_c(1-\alpha) = \alpha I_B + I_{co}$$

$$I_c = \frac{\alpha}{(1-\alpha)} I_B + \frac{I_{co}}{(1-\alpha)}$$

Characteristics of CB configuration of Transistor



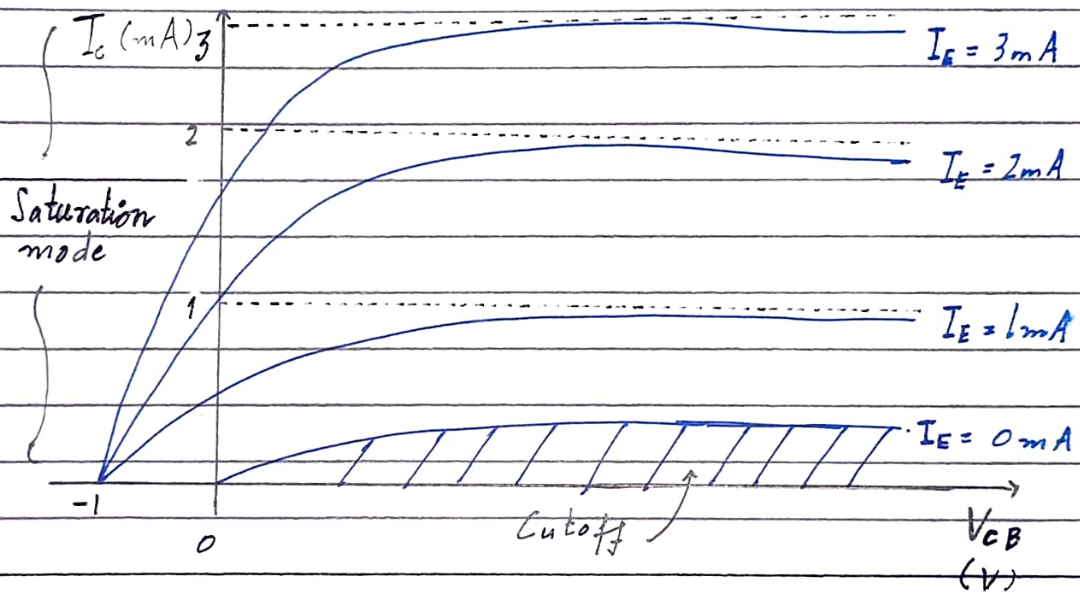
It is very useful to analyse the transistor based circuit.
It has two important characteristics ;
1. Input Characteristics
2. Output Characteristics

Input Characteristics
In CB configuration, the curve plotted between emitter current I_E and emitter base voltage V_{EB} at constant collector base voltage V_{CB} is called input characteristics.

When $V_{CB} = 0$ and the emitter base junction is forward biased the function behaves as a forward biased diode characteristics so that emitter current I_E increases exponentially with small increase in emitter base voltage V_{EB}.

When V_{CB} is increased keeping V_{EB} constant, the width of base region will be decrease. This affect the emitter current I_E increases sharply and the curve shift towards the left side. The emitter current increases exponentially with a small increase in emitter base voltage V_{EB}, as the input resistance $r_i = \frac{V_{EB}}{I_E}$ at constant V_{CB}.

→ Output Characteristics of CB configuration



In CB configuration, the curve plotted between collector current I_C and collector base voltage V_{CB} at constant emitter current I_E is called O/P characteristics.

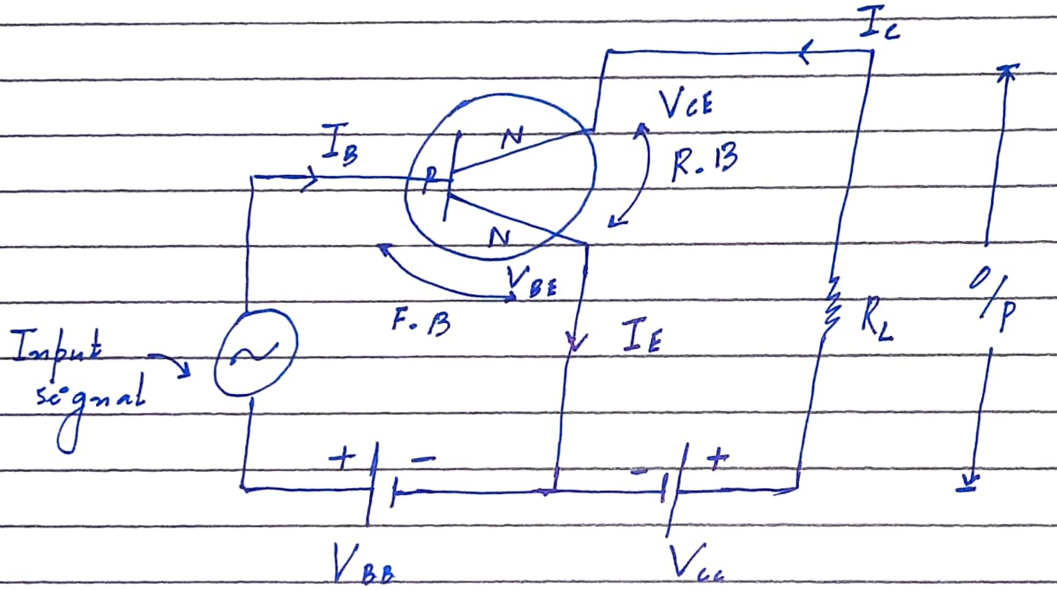
When collector base voltage is reverse biased, the I_C is most equal to the I_E. Therefore, transistor is always operated in active mode.

When V_{CB} becomes -ve, i.e collector base junction is forward biased and collector current I_C for a given emitter current I_B increases in saturation region. In this region, the collector current I_C does not depend upon the emitter current I_E, and when $I_E = 0$, collector current I_C is not equal to zero.

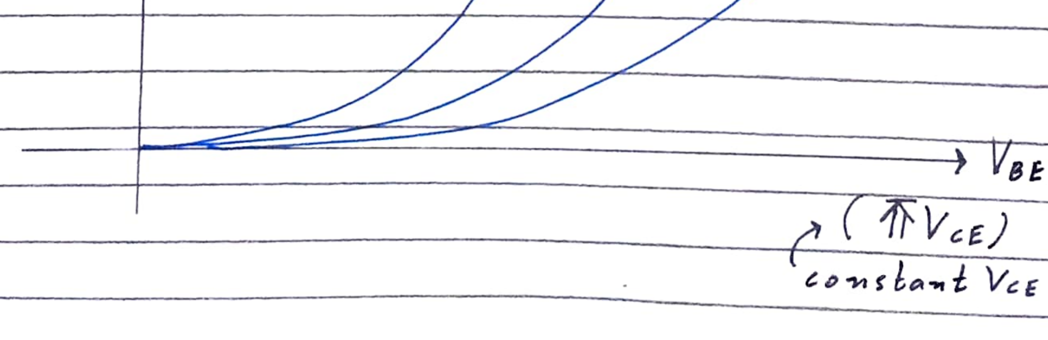
Although it's value is very small due to the reverse leakage current. I_{co} (temperature dependent current). Hence ~~the output resistance,~~

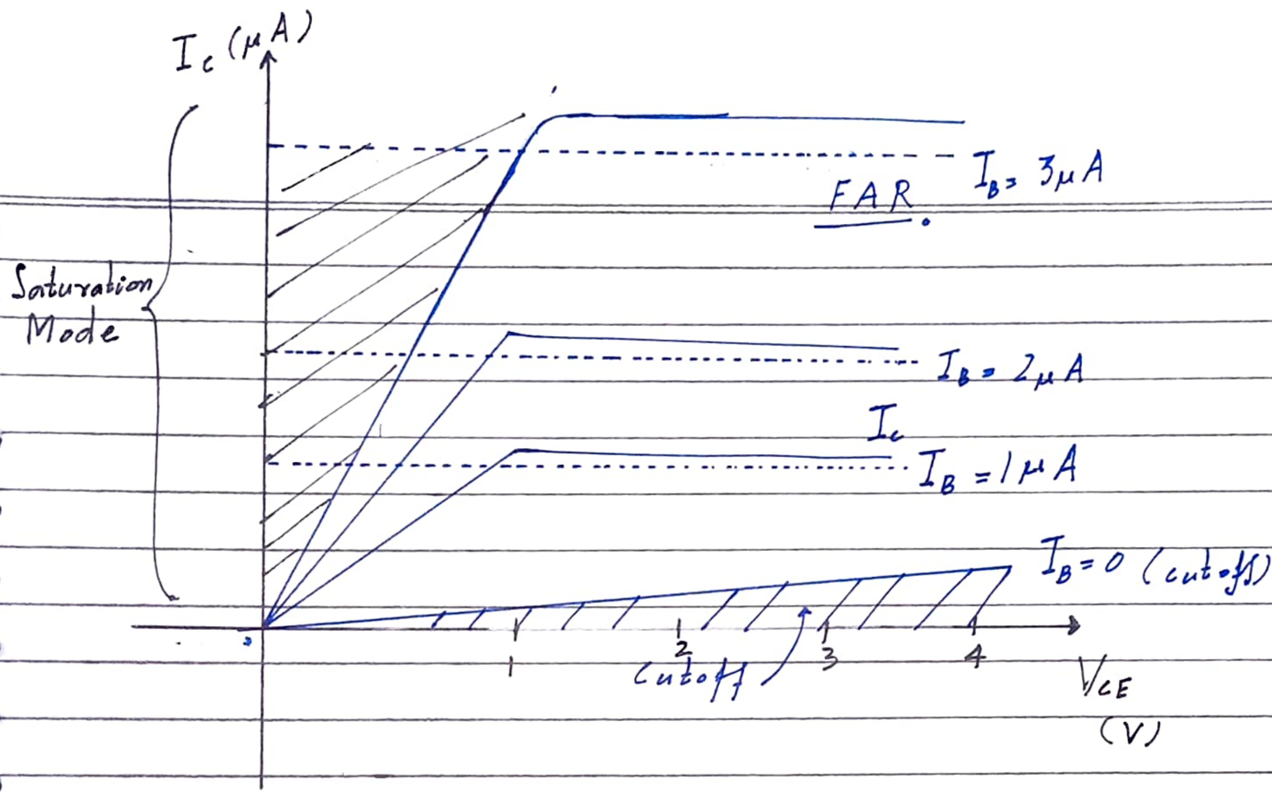
$$r_o = \frac{V_{CB}}{I_C} \text{ at constant } I_E.$$

2. Common Emitter Configuration of Transistor

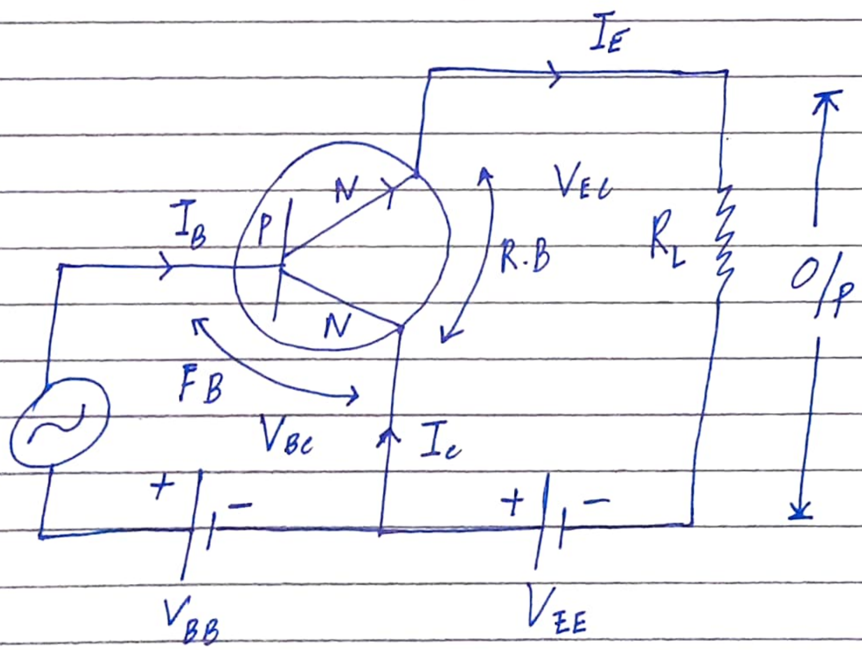


Amplification Factor : $\beta = \frac{I_C}{I_B}$ $\underbrace{(50-200)}_{\text{Commercial Range}}$





3. Common Collector Configuration



$$\text{Amplification Factor } (\gamma) = \frac{I_E}{I_B}$$

→ Relation between α and β;

$$I_E = I_B + I_c \quad (\div I_c)$$

$$I_E/I_c = I_B/I_c + 1$$

$$1/\alpha = 1/\beta + 1$$

$$\Rightarrow \boxed{\beta = \frac{\alpha}{1-\alpha}} < 1 \quad \text{and} \quad \boxed{\alpha = \frac{\beta}{\beta+1}} > 1$$

→ Relation b/w α, γ and β

$$\boxed{\gamma = \frac{1}{1-\alpha}}$$

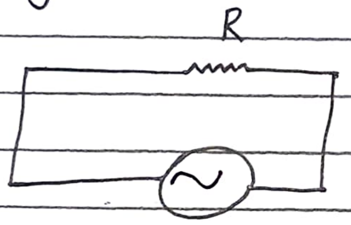
$$\boxed{\gamma = \frac{1}{1+\beta}}$$

Sinusoidal Steady State circuit
AC circuit

The closed path followed by the alternating current ~~is called AC circuit or the circuit in which~~ current and voltage vary sinusoidally are called AC circuit.

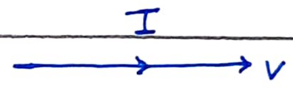
All AC circuits are madeup of combination of resistor, inductor and capacitor, in each case it is assumed that purely sinusoidally alternating voltage, $V = V_m \sin\omega t$ is applied to the circuit. The equation for the current, power and phase shift are developed in each case.

1. Purely Resistive Circuit



$V = V_m \sin\omega t$

$\therefore \quad i = \dfrac{V}{R}$



$i = \left(\dfrac{V_m}{R}\right)\sin\omega t$

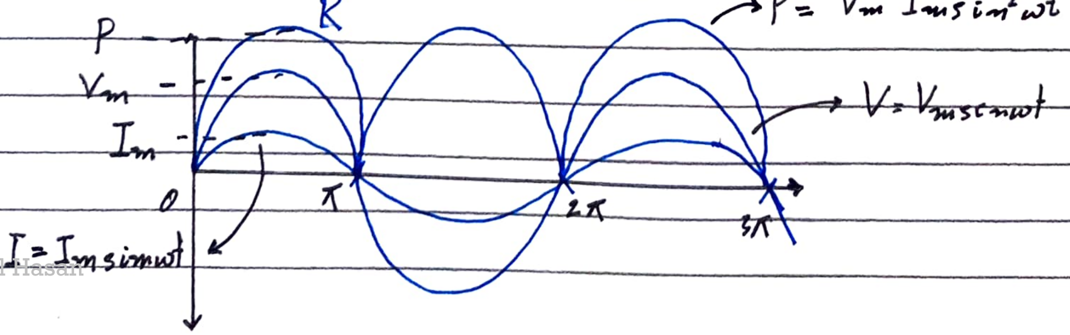
$\boxed{i = I_m \sin\omega t}$

$\phi = 0°$ (same phase)

for I_m,

$\theta = 90°$

$\Rightarrow \quad I_m = \dfrac{V_m}{R}$



Let the applied voltage:

$$V = V_m \sin\omega t \quad \text{— (1)}$$

According to Ohm's Law, we can find the equation for current, i.e

$$i = \frac{V}{R}$$

or $$i = \frac{V_m \sin\omega t}{R}$$

The instantaneous current i, is maximum when $\sin\omega t = 1$ or $\theta = 90°$ since the maximum value,

$$I_M = \frac{V_M}{R}.$$

Hence the instantaneous current is written by the equation

$$i = I_M \sin\omega t \quad \text{— (2)}$$

from eq (1) and (2) the alternating voltage and current has same frequency and phase therefore we can say that the alternating current and voltage are in same phase with each other.

Now,

$$p = V \times i$$

$$p = V_m \sin\omega t \times I_m \sin\omega t$$

$$p = V_m I_m \sin^2\omega t$$

$$p = V_m I_m \left(\frac{1-\cos 2\omega t}{2}\right)$$

$$p = \frac{V_m I_m}{2} - \underbrace{\frac{V_m I_m \cos 2\omega t}{2}}_{Av = 0}$$

$$p_{av} = \frac{V_m I_m}{2}$$

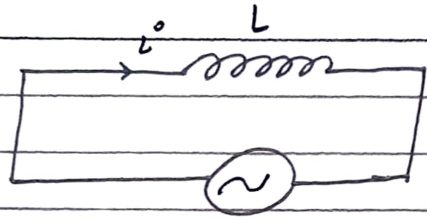
$$P_{av} = \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}} \rightarrow \text{rms values}$$

$$\therefore \boxed{P_{av} = VI}$$

The power consist of constant part $\frac{V_m \times I_m}{2}$ and fluctuating part with double frequency that of voltage and current. The average value of fluctuating part is always zero. Hence the average value of power.

$$P_{av} = \frac{V_m \times I_m}{2}$$

2. Purely Inductive Circuit



$V = V_m \sin\omega t$

$$e_L = -L\frac{di}{dt}$$

Applying KVL

$$\therefore \quad V + e_L = 0$$

$$V = L\frac{di}{dt}$$

$$V_m \sin\omega t = L\frac{di}{dt}$$

$$\int di = \frac{V_m}{L}\int \sin\omega t \, dt$$

$$i = \frac{V_m}{\omega L}(-\cos\omega t)$$

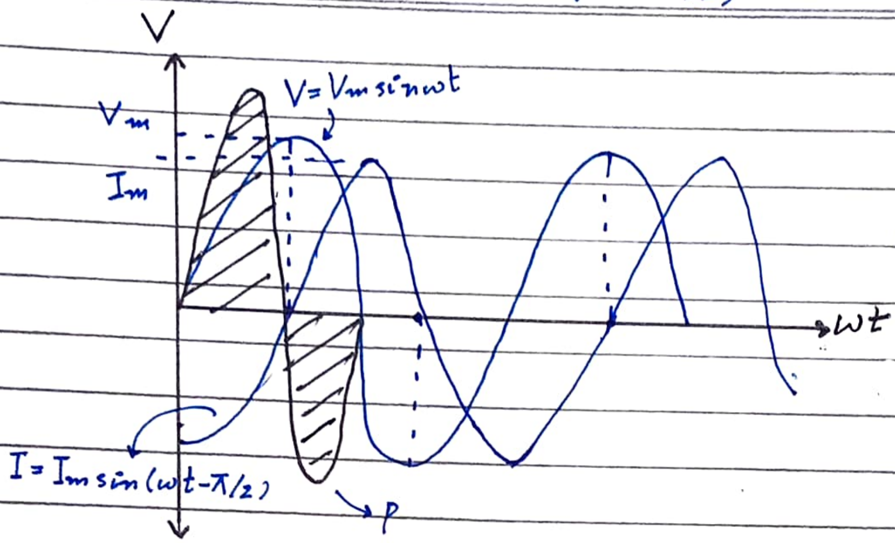
$$\boxed{i = \frac{V_m}{\omega L}\sin(\omega t - \pi/2)}$$

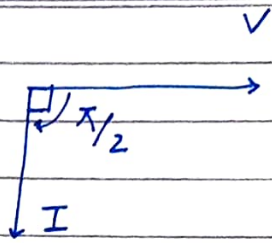
$X_L = \omega L$ Inductive reactance

$$\Rightarrow \quad i = \frac{V_m}{X_L}\sin(\omega t - \pi/2)$$

When, $\sin(\omega t - \pi/2) = 1$, $I_m = \dfrac{V_m}{X_L}$

$\therefore I = I_m \sin(\omega t - \pi/2)$



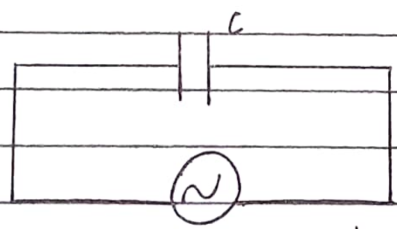


$$p = Vi$$
$$= V_m \sin\omega t \times I_m (\omega t - \pi/2)$$
$$= \frac{V_m I_m}{2} \sin 2\omega t$$

$$p_{av} = 0.$$

When power term is 'positive', energy gets stored in magnetic field due to the increasing current but due to the negative cycle in the power curve, this power is returned to supply.
Hence, the average power observed in pure inductive circuit is always 0.

Purely Capacitive Circuit



$V = V_m \sin \omega t$ ——— ①

$i = \dfrac{dq}{dt}$, and here

$Q = CV$

$\Rightarrow i = \dfrac{d(CV)}{dt}$

$$i = \frac{d(CV_m \sin \omega t)}{dt}$$

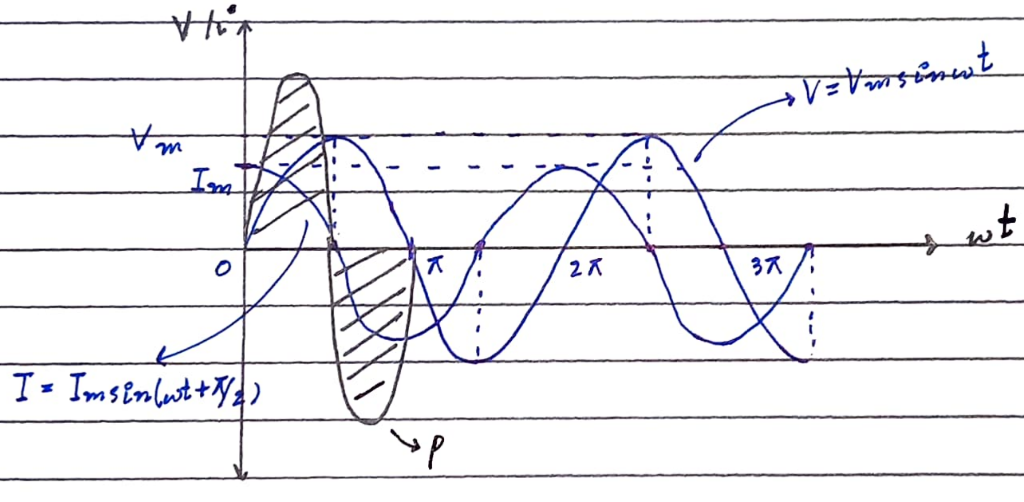
$$i = CV\omega \cos \omega t$$

$$i = \frac{V}{1/\omega C} \sin(\omega t + \pi/2)$$

$$\Rightarrow \quad i = \frac{V}{X_c} \sin(\omega t + \pi/2) \quad \text{—— (2)}$$

From eq (1) and (2), the alternating voltage and current have different phase i.e phase difference of $\pi/2$ rad or $90°$.

Therefore, the current is said to be leading ahead the voltage.



$$\therefore \quad P = VI$$

$$= V_m \sin \omega t \; I_m \sin(\omega t + \pi/2)$$

$$P_{avg} = \frac{V_m I_m}{2} \sin 2\omega t$$

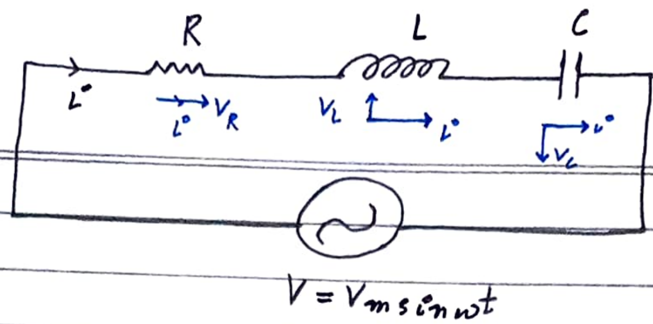
$$P_{avg} = 0$$

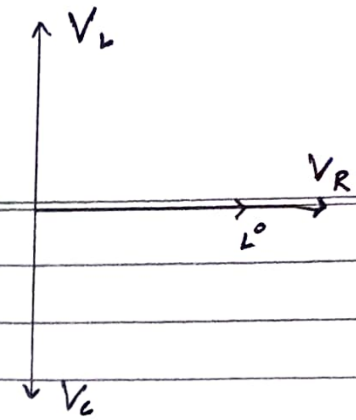
Hence, the power equation is purely sinosuidal of double frequency at an applied voltage.

Therefore, the average value of power over 1 complete cycle is 0.

When power curve is positive the energy gets stored in the capacitor during it's charging condition, while in the negative power curve, the stored energy is returned back to supply, during it's discharging.

Series RLC circuit





Series RLC circuit are extensively used in electrical circuit and hence their analysis is very important and it maybe noticed that all the AC quantities are vector quantities, they have both magnitude and direction. Either polar or rectangular these forms are extensively used in the analysis of AC circuits.

A series having resistance R, inductance L and capacitance C and applied voltage $V = V_m \sin \omega t$. Therefore, series RLC circuit on a current i. Due to this current, there are different voltage across the resistor, inductor and capacitor.

Now,

the voltage drop across resistance 'R';

$V_R = I.R$ (in phase with it)

the voltage drop across inductor 'L';

$V_L = I.X_L$ (leading by $\pi/2$)

and the voltag drop across capacitor 'C';

$V_C = I.X_C$ (lagging with current by an angle of $\pi/2$)

There are 4 voltages in series RLC circuit i.e V_R, V_L, V_C and resultant voltage V.

According to KVL, the resultant voltage

$V = V_R + V_L + V_C$

But, V_L and V_C are in opposite direction.

The resultant of V_L and V_C is the arithmetic difference between them.

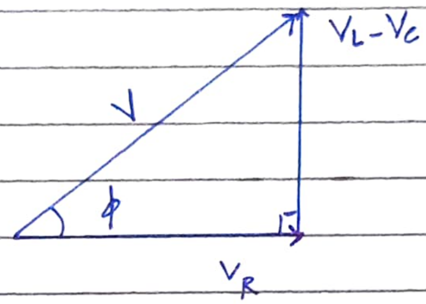
Therefore, three possible cases arises in series RLC circuit.

(i) $V_L > V_C$

(ii) $V_C > V_L$

(iii) $V_L = V_C$

Case 1: $V_L > V_C$



∴ Using Pythagoras Theorem

$\Rightarrow V^2 = V_R^2 + (V_L - V_C)^2$

$\Rightarrow V = \sqrt{V_R^2 + (V_L - V_C)^2}$

$$\therefore V = \sqrt{(IR)^2 + (IX_L - IX_C)^2}$$

$$V = I\sqrt{R^2 + (X_L - X_C)^2}$$

$$\Rightarrow V = IZ.$$

where, $Z = \sqrt{R^2 + (X_L - X_C)^2}$

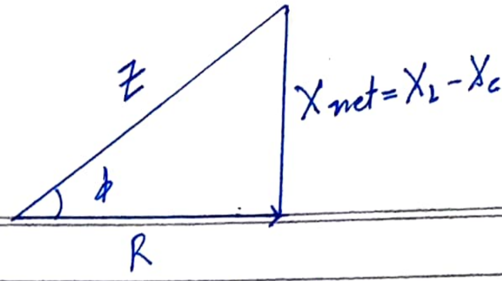
is known as the circuit impedence of series RLC circuit and

$X_{net} = X_L - X_C$ is called net reactance of series RLC circuit.

Impedence is defined as the total opposition offered to the current flow due to resistance, inductive reactance and capacitance reactance of the series RLC circuit. It is expressed in Ω.

$$\tan\phi = \frac{X_{net}}{R} = \frac{X_L - X_C}{R}$$

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) \rightarrow \text{positive}$$

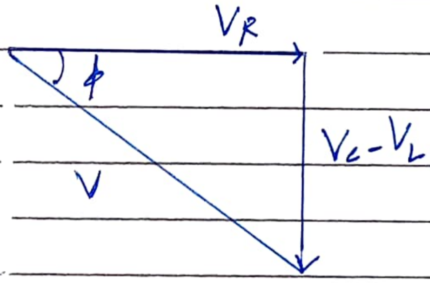


If $\omega L > \frac{1}{\omega C}$ and then the phase angle is positive, an then series RLC circuit becomes purely inductive circuit.

So, if ; $V = V_m \sin\omega t$, then the instantaneous current $I = I_m \sin(\omega t - \phi)$ because current lags behind the applied voltage by an angle ϕ.

(ii) $V_C > V_L$

Using Pythagores Theorem ;



$$V^2 = V_R^2 + (V_C - V_L)^2$$

$$V = \sqrt{V_R^2 + (V_C - V_L)^2}$$

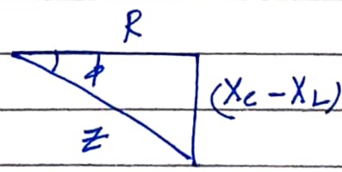
$$V = \sqrt{(IR)^2 + (IX_C - IX_L)^2}$$

$$V = I\sqrt{(R)^2 + (X_C - X_L)^2}$$

$$V = I \cdot Z$$

here

$$Z = \sqrt{R^2 + (X_C - X_L)^2} \longrightarrow \text{Impedence}$$



$$\tan\phi = \frac{X_{net}}{R} = \frac{X_C - X_L}{R}$$

$$\phi = \tan^{-1}\left(\frac{X_C - X_L}{R}\right) \rightarrow \text{negative}$$

$$\Rightarrow \frac{1}{\omega C} > \omega L$$

(متقل) $V_L = V_c$

This condition is the resonance.

A series RLC circuit is said to be in electrical resonance when the inductive ~~reason~~ reactance is equal to the capacitive reactance X_c and it's net reactance is 0. Resonance is defined as the condition in a circuit containing atleast one inductor and one capacitor and when the supply voltage and supply current are in same phase.

$$\therefore \quad Z = \sqrt{R^2 + (X_L - X_c)^2}$$

$$\text{As} \quad X_L = X_c$$

$$\Rightarrow \quad Z = R$$

$\therefore$ The total impedence of series RLC circuit at resonance condition $Z = R$.

It is also known as circuit impedence at resonance is called dynamic impedence.

$$X_L = X_c$$

$$\omega L = \frac{1}{\omega C}$$

$$\Rightarrow \omega^2 = \frac{1}{LC}$$

$$\omega_R = \sqrt{\frac{1}{LC}}$$

$$f_R = \frac{1}{2\pi\sqrt{LC}}$$

Resonance frequency

The frequency at which resonance occurs, is called resonance frequency. The series resonance effect maybe produced by the following methods.

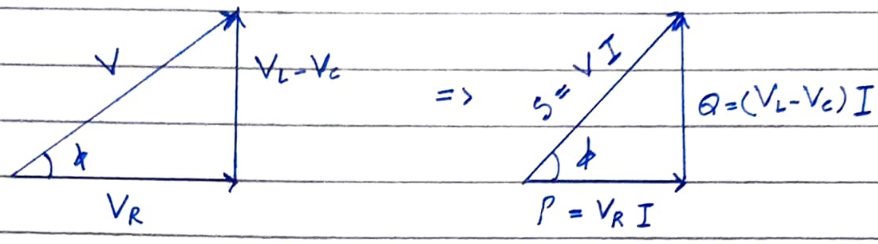
1. Varying frequency, keeping inductance and capacitance constant.

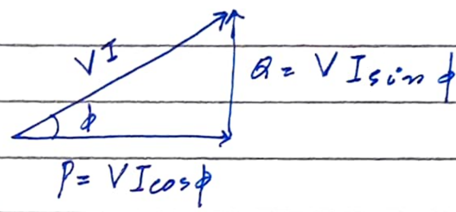
2. Varying either L or C for a given frequency.

Circuit current is maximum => $I_M = \dfrac{V_m}{R}$

Power in series RLC circuit.

$V_L > V_C$





For any condition $X_L > X_C$ or $X_L < X_C$ in general power can be expressed as :

P = voltage × component of current in phase with voltage

∴ Power consumed by series RLC circuit is :

$$P = VI\cos\phi$$

The power drawn by AC circuit can be of 3 types :

(i) Operand Power (S) : It is the product of RMS value of applied voltage and circuit current.

$$P = VI \text{ (volt-ampere)}$$

(ii) Active Power (P) : It is the power which is actually dissipated in the circuit resistance.

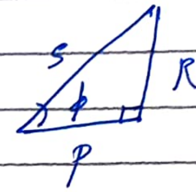
$$P = VI\cos\phi \text{ (watt).}$$

(iii) Reactive Power (Q): It is the power dissipated in inductive or capacitance reactance of the series RLC circuit and also defined as product of applied ~~voltage and reactive component of current.~~

$$P = VI\sin\phi \text{ (volt-ampere)}$$

From power Δ, these powers are related as

$$S^2 = P^2 + Q^2$$



→ Power Factor:

The power factor of series RLC circuit is a measure of it's effectiveness in utilizing the operand power drawn by the AC circuit or the ratio of active power to the operand power in series RLC circuit is defined as the power factor

$$\text{Power factor } (P_f) = \frac{\text{Active power}}{\text{Apparent Power}} = \frac{VI\cos\phi}{VI} = \cos\phi$$

$$\boxed{P_f = \cos\phi = R/Z}$$

At resonance;

$$\cos\phi = 1$$

$$\boxed{\phi = 0°}$$

→ Quality factor

The quality factor of series RLC circuit indicate how many times the potential difference across inductor or capacitor is greater than the applied voltage.

$$\text{Quality Factor } (Q) = \frac{\text{Voltage across } L \text{ or } C}{\text{Applied voltage}}$$

$\therefore \quad Q = \dfrac{I \cdot X_L}{I \cdot Z} = \dfrac{X_L}{Z}$ OR $\dfrac{I \cdot X_C}{Z} = \dfrac{X_C}{Z}$

$\therefore \quad Q = \dfrac{\omega_R L}{Z}$ or $\dfrac{1}{\omega_R C Z}$

$Q = \dfrac{\omega_R L}{R}$ $\left(\omega_R = \sqrt{\dfrac{1}{LC}} \rightarrow \text{Resonance frequency}\right)$

$Q = \dfrac{1}{R}\sqrt{\dfrac{L}{C}}$ $(0 \rightarrow 100) \rightarrow$ Range (Quality factor)

$(0 \rightarrow 1) \rightarrow$ Range (Power factor)

Q. 230 V, 50 Hz AC supply is applied to a coil of 0.06 H inductance and 2.5 Ω resistance connected in series with 6.8 μF capacitance.

Calculate

(i) Circuit Impedence

(ii) Circuit current

(iii) Phase angle between voltage and current

(iv) Total power consumed and power factor.

$\therefore$ (i) $L = 0.06\,H$, $C = 6.8 \times 10^{-6} F$

$f = 50\,Hz$

$\Rightarrow X_L = 2\pi f L = 2 \times \pi \times 50 \times 0.06 = 18.84\,\Omega$

$X_C = \dfrac{1}{2\pi f C} = \dfrac{1}{2 \times \pi \times 50 \times 6.8 \times 10^{-6}} = 468.102\,\Omega$

$\therefore Z = \sqrt{R^2 + (X_C - X_L)^2}$

$Z = \sqrt{(2.5)^2 + (449.262)^2} = 449.268\,\Omega$

(ii) $I_M = \dfrac{V_M}{Z} = \dfrac{230}{449.268} = 0.511\,A$ ans.

(iii) $\tan\phi = \frac{X_{net}}{R} = \frac{X_C - X_L}{R}$

$= \frac{449.262}{2.5}$

$\therefore \tan\phi = 179.704$

$\phi = \tan^{-1}(179.704)$

$\phi = 89.68°$

(iv) Power factor $= \cos\phi = \cos(89.68°) = 0.0058$

$\therefore P = VI\cos\phi$

$= 230 \times 0.511 \times \cos(89.68)$

$P = 0.656\ W$ Ans.

Q. A coil having a resistance 7Ω and inductance of 31.8 mH is connected to a 230 V and 50 Hz AC supply. Calculate

(i) Circuit current

(ii) Phase angle between voltage and current

(iii) Total power consumed

(iv) Power factor

$\therefore X_L = 2\pi f L = 2 \times \pi \times 50 \times 31.8 \times 10^{-3}$

$X_L = 9.990\ \Omega$

$\therefore Z = \sqrt{R^2 \quad X_L^2}$

$Z = \sqrt{7^2 + (9.99)^2}$

$Z = 12.19\ A$

(i) $I = \frac{V}{Z} = \frac{230}{12.19} = 18.86\ A$

(ii) $\tan\phi = \frac{X_L}{R} = 1.42$

$\phi = \tan^{-1}(1.42) = 54.8°$ Ans

(iii) and (iv)

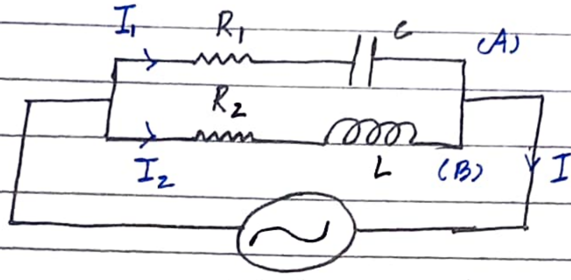
Power factor $\rightarrow$ $\cos\phi = \cos(54.8) = 0.576$

$\therefore P = VI\cos\phi$

$= 230 \times 18.86 \times \cos(54.8)$

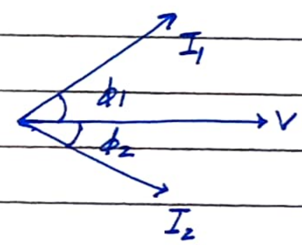
$= 2500.448$ W **Ans.**

→ Parallel RLC circuit



V = Vm sinωt

A parallel circuit consists of two branches connecting in parallel. Voltage is same for each branch but the branch current may differ in magnitude and phase depending on the branch impedence.



1. For branch A

$Z_1 = \sqrt{R^2 + X_C^2}$

$I_1 = \frac{V}{Z_1}$

$\phi_1 = \tan^{-1}\left(\frac{X_C}{R_1}\right)$

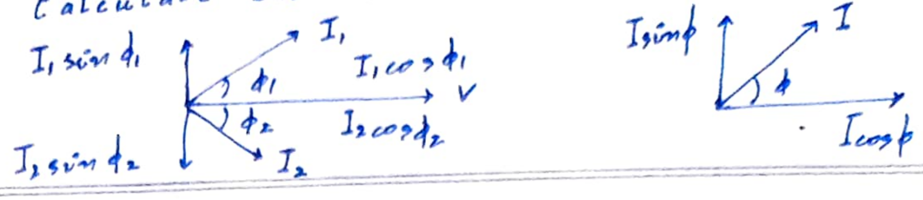
2. For branch B

$Z_2 = \sqrt{R^2 + X_L^2}$

$I_2 = V/Z_2$

$\phi_2 = \tan^{-1}\left(\frac{X_L}{R}\right)$

Calculate the resultant current I



$$\Rightarrow \quad I\cos\phi = I_1\cos\phi_1 + I_2\cos\phi_2$$
$$I\sin\phi = I_1\sin\phi_1 - I_2\sin\phi_2$$

$$I = \sqrt{(I\cos\phi)^2 + (I\sin\phi)^2}$$

$$\phi = \tan^{-1}\left(\frac{Y\text{-comp}}{X\text{comp}}\right) = \tan^{-1}\left(\frac{I\sin\phi}{I\cos\phi}\right)$$

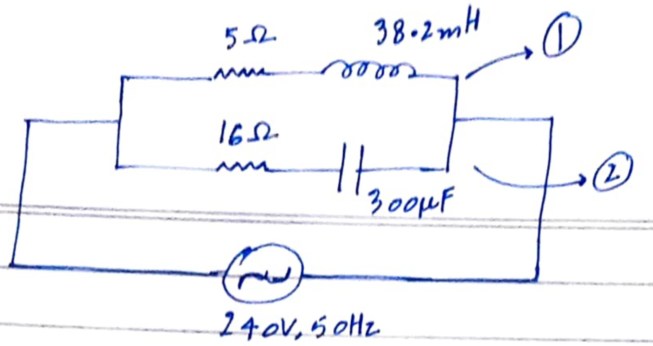
If $\tan\phi$ is +ve, then the resultant current I leads ahead the voltage and $\tan\phi$ is negative. Then the resultant current I lags behind the voltage.

$$\text{Power Factor} = \cos\phi = \frac{I\cos\phi}{I} = \frac{I_1\cos\phi_1 + I_2\cos\phi_2}{I}$$

$$\text{Quality Factor} = \frac{I_L}{I} \quad \text{or} \quad \frac{I_c}{I_L}$$

Q. A parallel circuit consists of two branches, one containing a coil a resistance $5\,\Omega$ and inductance 38.2 mH, the other non-inductive resistance $16\,\Omega$ in series with capacitor of $300\,\mu F$ capacitance. The circuit connected to a 240V and 50Hz AC supply. Determine;

(i) Current in each branch
(ii) Total resultant current
(iii) Phase angle between voltage and resultant current
(iv) Power factor and Quality factor



$\therefore$ For branch 1

$X_L = 2\pi f L = 2 \times \pi \times 50 \times 38.2 \times 10^{-3}$

$X_L = 12\ \Omega$

$\therefore$ $Z_1 = \sqrt{R^2 + X_L^2} = \sqrt{5^2 + 12^2}$

$Z_1 = 13\ \Omega$

$I_1 = \frac{V}{Z_1} = \frac{240}{13} = 18.46\ A$

$\therefore$ $\tan\phi_1 = \frac{X_L}{R} = 2.6$

$\phi_1 = \tan^{-1}(2.6) = 68.96°$

For branch 2

$X_c = \cancel{2\pi}\ \frac{1}{2\pi f C} = \frac{1}{2 \times \pi \times 50 \times 300 \times 10^{-6}}$

$X_c = 10.61\ \Omega$

$\therefore$ $Z_2 = \sqrt{R^2 + X_c^2} = \sqrt{16^2 + (10.61)^2} = 19.19\ \Omega$

$\Rightarrow$ $I_2 = \frac{V}{Z_2} = \frac{240}{Z_2} = \frac{240}{19.19} = 12.50 A$

$\therefore$ $\tan\phi_2 = \frac{X_c}{R} = 0.66$

$\phi_2 = \tan^{-1}(0.66) = 33.42°$ Ans

$$I\cos\phi = I_1\cos\phi_1 + I_2\cos\phi_2$$
$$= 18.46\cos(68.96) + 12.50\cos(0.66)$$
$$= 6.62 + 12.49$$
$$I\cos\phi = 19.11$$

$$I\sin\phi = I_1\sin\phi_1 - I_2\sin\phi_2$$
$$= 18.46\sin(68.96) + 12.50\sin(0.66)$$
$$= 17.22 + 0.14$$
$$I\sin\phi = 17.36$$

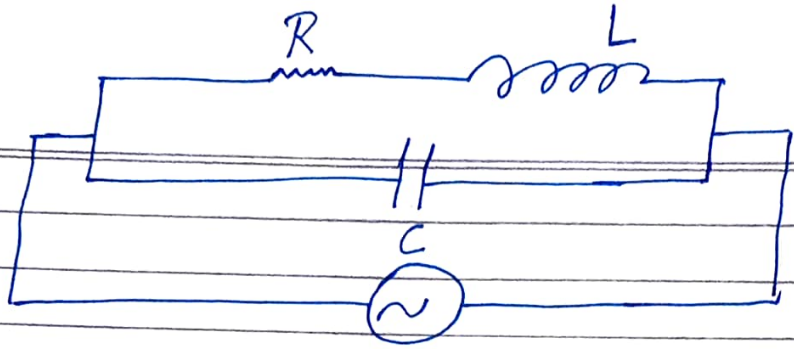
$$\Rightarrow \quad I = \sqrt{(I\cos\phi)^2 + (I\sin\phi)^2}$$
$$I = \sqrt{(19.11)^2 + (17.36)^2}$$
$$I = 25.81\,A$$

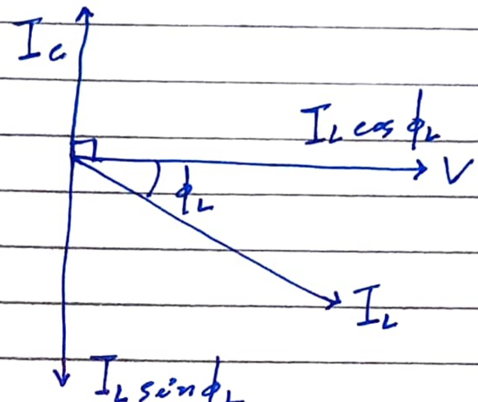
$$\therefore \quad \tan\phi = \frac{I\sin\phi}{I\cos\phi} = \frac{17.36}{19.11} = 0.908$$
$$\phi = \tan^{-1}(0.908) = 42.23^\circ$$ Ans.

$$\therefore \quad \text{Power factor} = \cos\phi = \frac{I\cos\phi}{I} = \frac{19.11}{25.81} = 0.74$$

$$\text{Quality factor} = \frac{I_L}{I} = \frac{18.46}{25.81} = 0.715$$ Ans.

Resonance in Parallel RLC circuit;





Consider a practical case of parallel RLC circuit consisting of a resistance R in series with inductor L and both connected parallel to a capacitor C across the AC voltage $V = V_m \sin\omega t$, the current flows in inductive branch I_L and capacitive branch I_c;

$$I_L = V/Z_L$$

$$Z_L = \sqrt{R^2 + X_L^2}$$

$$\phi_L = \tan^{-1}\left(\frac{X_L}{R}\right)$$

and $$I_c = \frac{V}{X_c}$$

the current I_L lags behind the applied voltage V by an angle ϕ_L while the current I_c leads, the applied voltage V by an angle of 90°. Therefore, such a circuit is in resonance when the reactive component of line current is 0 i.e

$$I_c - I_L \sin\phi = 0$$

$$I_c = I_L \sin\phi$$

$$\frac{V}{X_c} = \frac{V}{X_L}\sin\phi$$

$$\omega C = \frac{1}{\sqrt{R^2+X_L^2}}\sin\phi$$

$$\omega C = \frac{1}{\sqrt{R^2+X_L^2}} \times \frac{X_L}{Z_L}$$

$$Z_L^2 = X_L \cdot X_c$$

$$R^2+\omega^2 L^2 = X_L \cdot X_c$$

$$R^2 + X_L^2 = \frac{L}{C}$$

$$X_L^2 = \frac{L}{C} - R^2$$

$$\boxed{\omega = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L}}}$$

$$\omega = \frac{1}{2\pi}\sqrt{\frac{1}{LC}} \;;\; \text{if } \frac{R^2}{L^2} << \frac{1}{LC}$$

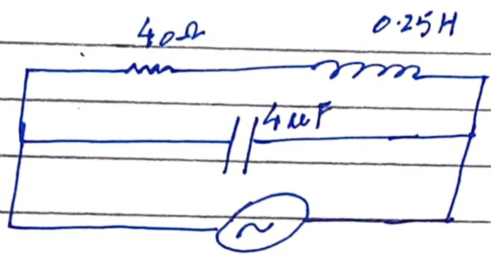
then,

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}$$

If resistance R of the parallel RLC circuit is very small or $\frac{R^2}{L^2} <<< \frac{1}{LC}$. then the resonance frequency of parallel RLC circuit $f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}$ which is same as resonance frequency of series RLC circuit.

Quality factor : $\frac{I_L}{I_L \cos\phi_L} = \frac{1}{\cos\phi_L} = \sec\phi_L$

Q. A circuit consists of $4\mu F$ capacitor in parallel with coil of resistance 40Ω and inductance 0.25 H. If a voltage applied to the circuit and parallel AC circuit is 230 V at resonance frequency f_0. Calculate
(i) current in each branch
(ii) resultant current
(iii) phase angle between voltage and current
(iv) Quality factor and power factor.



$X_L = 2\pi f L$
$= 2\times\pi\times 157.1 \times 0.25$
$= 246.77\ \Omega$

First calculate f_0, the put f_0 in X_L

and $f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$

$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{0.25\times4\times10^{-6}} - \frac{40^2}{(0.025)^2}} = 157.104\ Hz$

$I_L = \frac{V}{Z_L} = \frac{230}{\sqrt{40^2+246.77^2}} = 0.960\ A$

$I_c = \frac{V}{X_c} = 230\times2\times\pi\times157.104\times4\times10^{-6} = 0.908\ A$

(ii) $I = I_L \cos\phi_L = 0.960 \cos\left(\frac{246.77}{40}\right) = 0.147\ A$ Ans

(iii) At resonance:

$\phi = 0°$

Power factor $= \cos\phi = 1$

(iv) Quality factor $= \sec\phi_L = \sec(80.78) = 6.241$

Ans

These handwritten notes are of ESC-S101 taught to us by Prof. Om Pal, compiled and organized chapter-wise to help our juniors. We hope they make your prep a bit easier.

— **Saksham Nigam** and **Misbahul Hasan** (B.Tech. CSE(2024-28))