Network Theory

1. Circuit Theory Analysis – Mesh Analysis and Nodal Analysis.
2. Star-Delta Transformation
3. Network Theorem – Superposition Theorem, Norton Theorem and Maximum Power transfer Theorem.

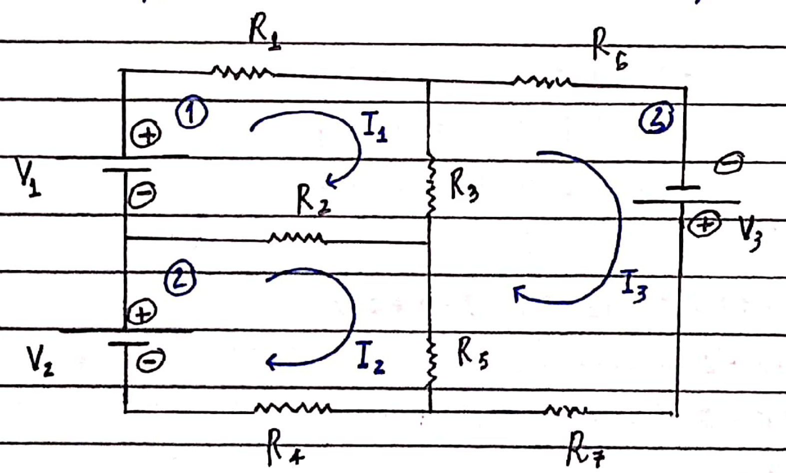
→ Kirchoff's Voltage Law: The algebraic sum of voltage drop in any closed path in a specified direction is equal to 0.

→ Kirchoff's Current Law: The algebraic sum of incoming and outgoing current in a junction is 0.

1. Mesh Analysis:

Mesh or loop analysis is based on Kirchoff's voltage law, it is used to find out unknown current, voltage in the mesh.

In this method, each mesh is assigned a seperate mesh current and KVL is applied to write the mesh equation.



$R_6 I_3 + R_7 I_3 + R_5 (I_3 - I_2) +$
$R_3 (I_3 - I_1) = 0$

$-I_1 R_3 - I_2 R_5 + I_3 (R_3 + R_5 + R_6 + R_7)$
$= V_3 \quad — (3)$

$I_1 R_1 + R_3 (I_1 - I_3) + R_2 (I_1 - I_2) = V_1$

$I_1 (R_1 + R_2 + R_3) - I_2 R_2 - I_3 R_3 = V_1 \quad — (1)$

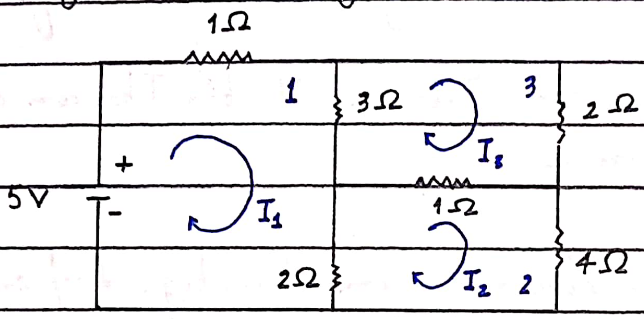
$R_2 (I_2 - I_1) + R_5 (I_2 - I_3) + R_4 I_2 = V_2$

$-R_2 I_1 + I_2 (R_2 + R_5 + R_4) - I_3 R_5 = V_2 \quad — (2)$

©Misbahul Hasan

Scanned with OKEN Scanner

Q. Find the current I_1, I_2 and I_3 of the given network using mesh analysis.



$5I_1 + 3(I_1 - I_3) + 2(I_1 - I_2) = 5$

$I_1(10) - I_2 . 2 + 5I_1 = 5$

$10I_1 - 2I_2 - 3I_3 = 5$ — (1)

$3(I_3 - I_1) + 1(I_3 - I_2) + 2I_3 = 0$

$6I_3 - 3I_1 - I_2 = 0$ — (3)

$1(I_2 - I_3) + 4I_2 + 2(I_2 - I_1) = 0$

$-2I_1 + 7I_2 - I_3 = 0$ — (2)

Arranging the mesh equations into matrix form and using Crammer's rule.

$[I][R] = [V]$

$$\Rightarrow \begin{bmatrix} 6 & -2 & -3 \\ -2 & 7 & -1 \\ -3 & -1 & 6 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ 0 \end{bmatrix}$$

$\therefore \Delta = 6(42-1) + 2(-12+3) - 3(2+21)$

$= 6(41) + 2(-15) - 3(23)$

$= 246 - 30 - 69$

$= 147$

$$\therefore \begin{bmatrix} 5 & -2 & -3 \\ 0 & 7 & -1 \\ 0 & -1 & 6 \end{bmatrix}, \quad \Delta_1 = 5(41) + 2(0) - 3(0)$$

$= 205$

©Misbahul Hasan
Scanned with OKEN Scanner

$$\Delta_2 = \begin{vmatrix} 6 & 5 & -3 \\ -2 & 0 & -1 \\ -3 & 0 & 6 \end{vmatrix} = 6(0) - 5(-15) - 3(0) = +75$$

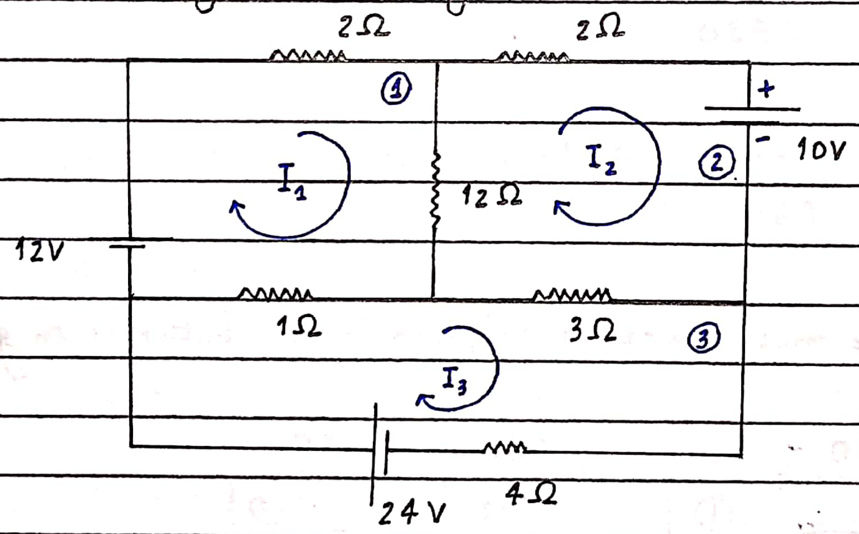
$$\Delta_3 = \begin{vmatrix} 6 & -2 & 5 \\ -2 & 7 & 0 \\ -3 & -1 & 0 \end{vmatrix} = 6(0) + 2(0) - 5(23) = -115$$

$$\Rightarrow \quad I_1 = \frac{\Delta_1}{\Delta} = \frac{205}{147} = 1.39\,A$$

$$I_2 = \frac{\Delta_2}{\Delta} = \frac{75}{147} = 0.51\,A$$

$$I_3 = \frac{\Delta_3}{\Delta} = \frac{-115}{147} = -0.78\,A \text{ ans}$$

Q. Determine the branch current in 4Ω resistor in the given network using mesh analysis.



$$\therefore \quad 2I_1 + 12(I_1 - I_2) + 1(I_1 - I_3) = 12$$

$$15I_1 - 12I_2 - I_3 = 12 \quad \text{— (1)}$$

$$2I_2 + 12(I_2 - I_1) + 3(I_2 - I_3) = -10$$

$$-12I_1 + 17I_2 - 3I_3 = -10 \quad \text{— (2)}$$

$$1(I_3 - I_1) + 3(I_3 - I_2) + 4I_3 = 24$$

$$-I_1 - 3I_2 + 8I_3 = 24 \quad \text{— (3)}$$

©Misbahul Hasan
Scanned with OKEN Scanner

Arranging mesh equations into matrix form and using crammer's rule.

$\therefore \quad [R][I] = [V]$

$$\begin{bmatrix} 15 & -12 & -1 \\ -12 & 17 & -3 \\ -1 & -8 & 8 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 12 \\ -10 \\ 24 \end{bmatrix}$$

$$\therefore \quad \Delta = \begin{vmatrix} 15 & -12 & -1 \\ -12 & 17 & -3 \\ -1 & -3 & 8 \end{vmatrix} = 15(17.8-9)+12(-96-3)-1(36+1$$

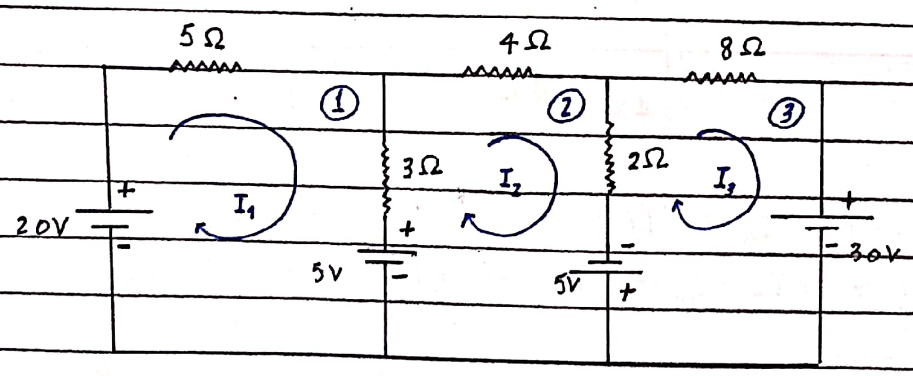
$\therefore \quad \Delta = 664$

$$\Delta_3 = \begin{vmatrix} 15 & -12 & 12 \\ -12 & 17 & -10 \\ -1 & -3 & 24 \end{vmatrix} = 15(17.24-30)+12(-12.24-10)+12(36+17)$$

$\Delta_3 = 2730$

$\Rightarrow \quad I_3 = \dfrac{2730}{664} = 4.11\ A$ ans

Q. Determine the mesh current supplied by the batteries in given network.



$5I_1 + 3(I_1 - I_2) = 20 - 5$

$8I_1 - 3I_2 = 15$ — (1)

$4I_2 + 3(I_2 - I_1) + 2(I_2 - I_3) = 5+5+5$

$-3I_1 + 9I_2 - 2I_3 = 15$ — (2)

$8I_3 + 2(I_3 - I_2) = -30 - 5$

$-2I_2 + 20I_3 = -35$ — (3)

©Misbahul Hasan

Scanned with OKEN Scanner

Arranging the mesh equations into matrix form and using Crammer's Rule.

$$\begin{bmatrix} 8 & -3 & 0 \\ -3 & 9 & -2 \\ 0 & -2 & 10 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 15 \\ 15 \\ -35 \end{bmatrix}$$

$$\therefore \quad \Delta = 8(90-4)+3(-30)$$
$$= 8(86)-90$$
$$= 598 \text{ . ans}$$

$$\Delta_1 = \begin{vmatrix} 15 & -3 & 0 \\ 15 & 9 & -2 \\ -35 & -2 & 10 \end{vmatrix} = 15(86)+3(150-70) = 1050$$

$$\Delta_2 = \begin{vmatrix} 8 & 15 & 0 \\ -3 & 15 & -2 \\ 0 & -35 & 10 \end{vmatrix} = 8(150-70)-15(-30)$$
$$= 8(80)+450$$
$$= 640+450 = 1090$$

$$\Delta_3 = \begin{vmatrix} 8 & -35 & 15 \\ -3 & 9 & 15 \\ 0 & -2 & -35 \end{vmatrix} = 8(9(-35)-30)+35(3\times35)+15(6)$$
$$= 1005$$

$$\Rightarrow \quad I_1 = \frac{\Delta_1}{\Delta} = \frac{1050}{598} = 1.755 \text{ A}$$

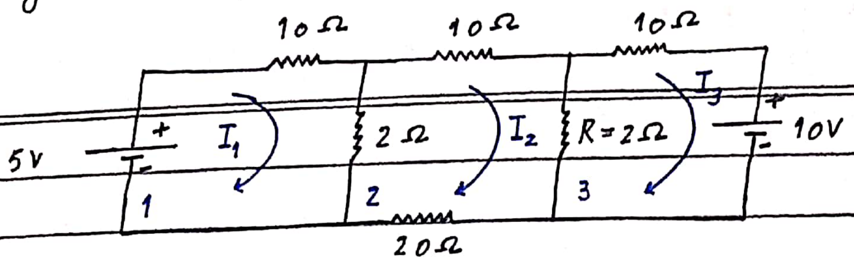
$$I_2 = \frac{\Delta_2}{\Delta} = \frac{1090}{598} = 1.822 \text{ A}$$

$$I_3 = \frac{\Delta_3}{\Delta} = \frac{1005}{598} = 1.68 \text{ A}$$ ans.

©Misbahul Hasan

Scanned with OKEN Scanner

Q. Find the voltage across the resistor R
using Mesh Analysis.



Applying mesh analysis in mesh 1

$$-10 I_1 - 2(I_1 - I_2) + 5 = 0$$

$$-12 I_1 + 2 I_2 = -5$$

$$12 I_1 - 2 I_2 = 5 \quad \text{——} \quad (1)$$

Applying mesh analysis in mesh 2

$$-10 I_2 - 2(I_2 - I_3) - 20 I_2 - 2(I_2 - I_1) = 0$$

$$-34 I_2 + 2 I_3 + 2 I_1 = 0$$

$$I_1 - 17 I_2 + I_3 = 0 \quad \text{——} \quad (2)$$

Applying mesh analysis in mesh 3

$$-10 - 2(I_3 - I_2) - 10 I_3 = 0$$

$$+2 I_2 - 12 I_3 = 10$$

$$I_2 - 6 I_3 = 5 \quad \text{——} \quad (3)$$

Arranging mesh equations into matrix form and applying crammer's rule.

$$\therefore \begin{bmatrix} 12 & -2 & 0 \\ 1 & -17 & 1 \\ 0 & 1 & -6 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ 5 \end{bmatrix}$$

$$\therefore \quad \Delta = \begin{vmatrix} 12 & -2 & 0 \\ 1 & -17 & 1 \\ 0 & 1 & -6 \end{vmatrix} = 12(17(6) - 1) + 2(-6)$$

$$12(101) - 12$$

$$= 1200.$$

©Misbahul Hasan
Scanned with OKEN Scanner

$$\Delta_2 = \begin{vmatrix} 12 & 5 & 0 \\ 1 & 0 & 1 \\ 0 & 5 & -6 \end{vmatrix} = 12(-5) - 5(-6) = -60 + 30 = -30$$

$$\Delta_3 = \begin{vmatrix} 12 & -2 & 5 \\ 1 & -17 & 0 \\ 0 & 1 & 5 \end{vmatrix} = 12((-17)(5)) + 2(5) + 5(1) = -1005$$

$$\therefore \quad I_2 = \frac{\Delta_2}{\Delta} = \frac{-30}{1200} = -0.025$$

$$I_3 = \frac{\Delta_3}{\Delta} = \frac{-1005}{1200} = -0.8375$$

$\therefore$ The current through $R = 2\Omega$;

$$I_{2\Omega} = I_2 - I_3$$
$$= (-0.025) - (-0.8375)$$
$$I_{2\Omega} = 0.8125 \text{ A}$$ ans.

$\Rightarrow$ Voltage across 2Ω resistor;

$$V_{2\Omega} = I_{2\Omega} \times 2$$
$$= 0.8125 \times 2$$
$$V_{2\Omega} = 1.625 \text{ V}$$ Ans.

Nodal Analysis (Based on KCL)

Nodal Analysis is based on KCL, in this method we define the voltage across each node as an independent.

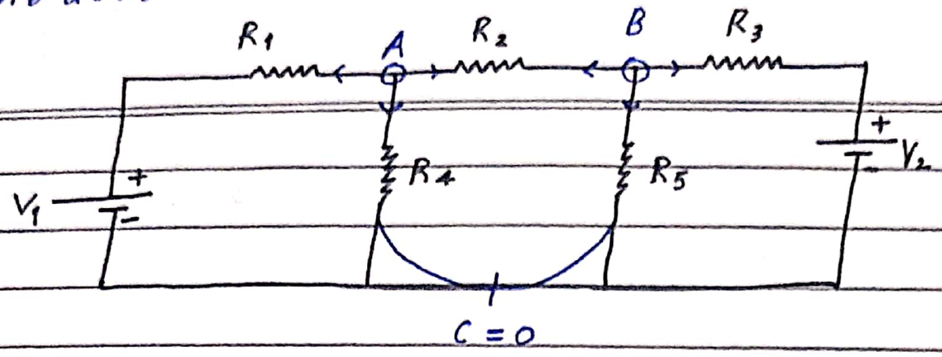
For the application every node junction in the network where three or more branches meet is regarded as a node. One of these is regarded as a reference node or zero potential node.

Hence, the no. of simultaneous equations to be solved

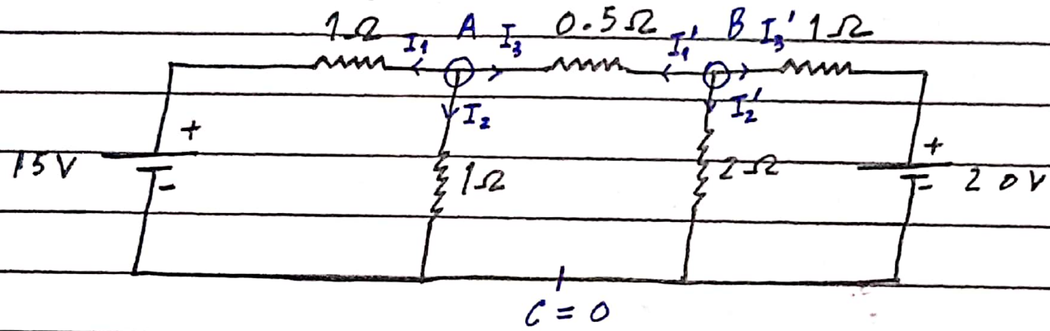
©Misbahul Hasan

Scanned with OKEN Scanner

becomes (n-1) where n is the no. of independent nodes.



Q. Find the branch currents of the given network using nodal analysis.



Let 'A' and 'B' be two node junctions and 'C' be the reference node.

Applying KCL for node A

$$\frac{V_A - 15}{1} + \frac{V_A - 0}{1} + \frac{V_A - V_B}{0.5} = 0$$

$$\therefore \quad 2V_A - 15 + 2V_A - 2V_B = 0$$

$$4V_A - 2V_B = 15 \quad \text{—} \quad (1)$$

Now, applying KCL for node B

$$\frac{V_B - 20}{1} + \frac{V_B - 0}{2} + \frac{V_B - V_A}{0.5} = 0$$

$$\frac{3}{2}V_B - 20 + 2V_B - 2V_A = 0$$

$$7V_B - 4V_A = 20 \times 2$$

$$7V_B - 4V_A = 40 \quad \text{—} \quad (2)$$

©Misbahul Hasan
Scanned with OKEN Scanner

Adding ① and ②

$\therefore \quad 4V_A - 2V_B = 15$

$\quad\;\; 7V_B - 4V_A = 40$

$\quad\;\; 5V_B = 55$

$\therefore \quad V_B = \frac{55}{5} = 11\,V$

$\Rightarrow \quad 4V_A - 22 = 15$

$\quad\;\; V_A = 9.25\,V$

$\therefore \quad I_1 = 9.25 - 15 = -5.75\,A$

$I_2 = 9.25\,A$

$I_3 = (9.25 - 11)2 = -3.5\,A$

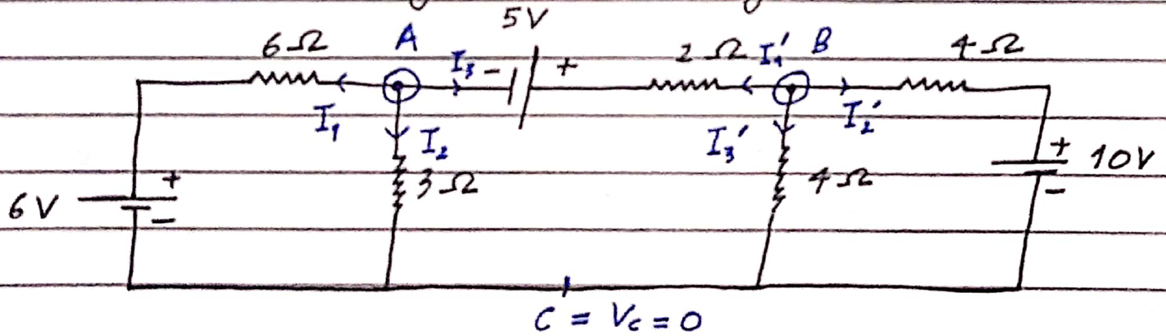
$I_2' = 5.5\,A$

$I_3' = 11 - 20 = -9\,A$

$I_1' = (11 - 9.25) \times 2 = 3.5$

$I_{0.5\Omega} = 3.5 - (-3.5) = 7\,A$ Ans.

Q. Find the branch current in the given network using nodal analysis.



Let 'A' and 'B' be two node junctions with potential 'V_A' and 'V_B' respectively and 'C' be reference potential node.

$\therefore$ Applying KCL on node (A)

$$\frac{V_A - 6}{6} + \frac{V_A - 0}{3} + \frac{V_A + 5 - V_B}{2} = 0$$

©Misbahul Hasan

Scanned with OKEN Scanner

$$V_A - 6 + 2V_A + 3V_A + 15 - 3V_B = 0$$
$$6V_A - 3V_B = -9$$
$$2V_A - V_B = -3 \quad \text{———} \quad (1)$$

Now, applying KCL on node 'B';

$$\frac{V_B - 5 - V_A}{2} + \frac{V_B}{4} + \frac{V_B - 10}{4} = 0$$

$$2V_B - 10 - 2V_A + 2V_B - 10 = 0$$
$$4V_B - 2V_A = 20 \quad \text{———} \quad (2)$$

$\therefore$ Adding (1) and (2)

$$\cancel{2V_A} - V_B = -3$$
$$4V_B - \cancel{2V_A} = 20$$
$$\overline{3V_B \quad = 17}$$

$$V_B = \frac{17}{3} = 5.66\,V$$

$\therefore$ $2V_A = -3 + V_B$

$$V_A = \frac{-3 + 5.66}{2} = 1.33\,V$$

$\Rightarrow$ $I_1 = \frac{V_A - 6}{6} = \frac{1.33 - 6}{6} = -0.77\,A$

$$I_2 = \frac{V_A}{3} = \frac{1.33}{3} = 0.44\,A$$

$$I_3 = \frac{V_A + 5 - V_B}{2} = \frac{1.33 + 5 - 5.66}{2} = 0.335\,A$$

©Misbahul Hasan

Scanned with OKEN Scanner

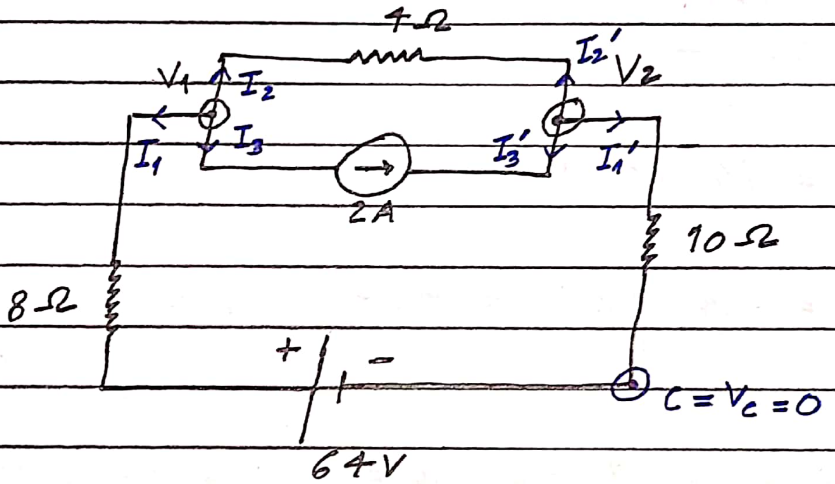
$$I_1' = \frac{V_B - 5 - V_A}{2} = \frac{5.66 - 5 - 1.33}{2} = -0.335$$

$$I_2' = \frac{V_B - 10}{4} = \frac{5.66 - 10}{4} = -1.085 A$$

$$I_3' = \frac{V_B}{4} = \frac{5.66}{4} = 1.415 \ A$$

$\therefore \quad I_{AB} = I_1 - I_1' = 0.335 - (-0.355) = 0.67$ Ans.

Q. Using Nodal Analysis, calculate V_1 and V_2.



Let 'C' be the reference node.

$$\begin{array}{r|l} 2 & 10, 4 \\ 2 & 5 \\ 5 & \end{array}$$

Applying KCL on V_1.

$$\frac{V_1 - 64}{8} + \frac{V_1 - V_2}{4} + 2 = 0$$

$$V_1 - 64 + 2V_1 - 2V_2 = -16$$

$$3V_1 - 2V_2 = 48 \text{ ——— } (1)$$

Applying KCL on V_2.

$$\frac{V_2}{10} + \frac{V_2 - V_1}{4} - 2 = 0$$

$$2V_2 + 5V_2 - 5V_1 = 40$$

$$7V_2 - 5V_1 = 40 \text{ ——— } (2)$$

©Misbahul Hasan

Scanned with OKEN Scanner

Multiplying eq ① by 5

$15V_1 - 10V_2 = 240$ — ③

Multiplying eq ② by 3

$21V_2 - 15V_1 = 120$ — ④

Adding ③ and ④

$$\begin{aligned} \cancel{15V_1} - 10V_2 &= 240 \\ 21V_2 - \cancel{15V_1} &= 120 \\ \hline 11V_2 &= 120 \end{aligned}$$

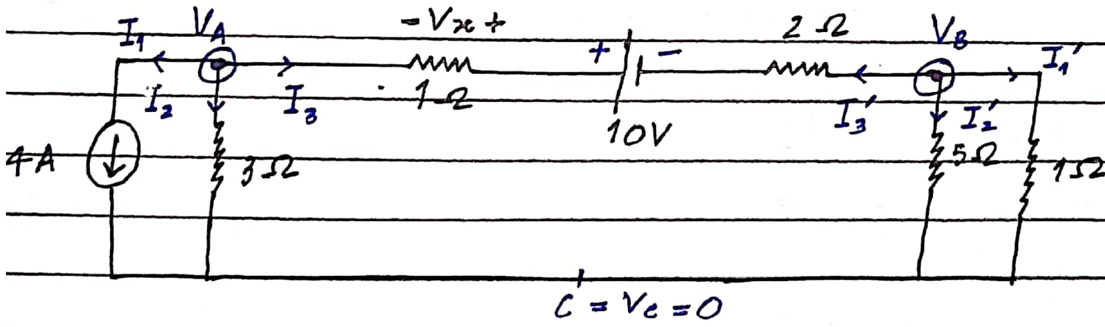
$V_2 = 10.90 \text{ V}$

and

$V_1 = \frac{48 + 2V_2}{3} = \frac{48 + 21.81}{3} = 23.27\text{V}$

Ans.

Q. Calculate the current through 5Ω also find V_x.



Let 'A' and 'B' be two nodes with potentials as 'V_A' and 'V_B' respectively.
'C' be the reference node.

Applying KCL on node 'A'

$4 + \frac{V_A}{3} + \frac{V_A - 10 - V_B}{3} = 0$

$2V_A - V_B = -2$ — ①

©Misbahul Hasan

Scanned with OKEN Scanner

Applying KCL on node 'B'

$\therefore \quad V_B + \frac{V_B}{5} + \frac{V_B + 10 - V_A}{3} = 0$

$15V_B + 3V_B + 5V_B + 50 - 5V_A = 0$

$23V_B - 5V_A = -50 \quad \text{--- (2)}$

$\therefore \quad (1) \times 5 + (2) \times 2$

$\Rightarrow \quad 10V_A - 5V_B = -10$

$46V_B - 10V_A = -100$

$41V_B = -110$

$V_B = \frac{-110}{41} = -2.68\,V$

$\therefore \quad V_A = \frac{-2 + V_B}{2} = -2.34\,V$

$\therefore \quad I_{3\Omega} = \frac{V_B}{5} = \frac{-2.68}{5} = -0.536\,A$ ans.

and $I_3 = \frac{V_A - 10 - V_B}{3} = \frac{-2.34 - 10 + 2.68}{3} = -3.22\,A$

$I_3' = \frac{V_B + 10 - V_A}{3} = \frac{-2.68 + 10 + 2.3}{3} = 3.22\,A$

$I_{1\Omega} = I_3' - I_3$

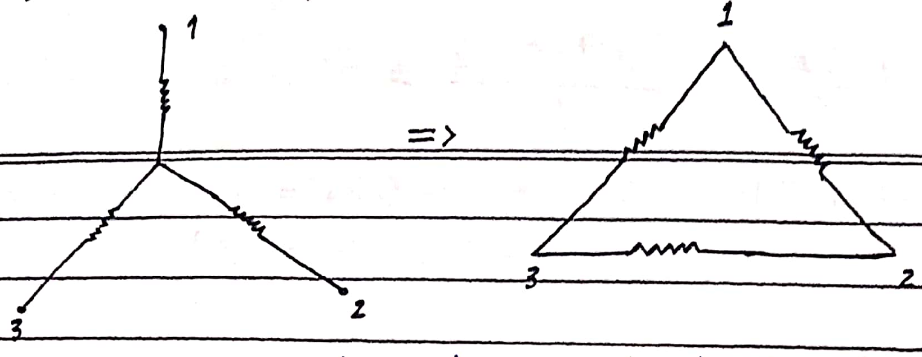
$\therefore \quad I_{1\Omega} = 3.22 - (-3.22) = 6.44\,A$

$\Rightarrow \quad V_x = 1 \times 6.44 = 6.44\,V$ Ans.

©Misbahul Hasan

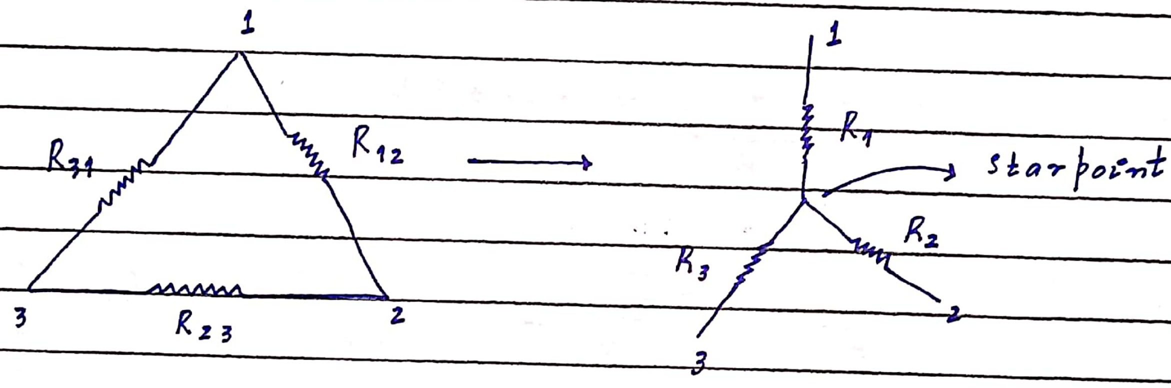
Scanned with OKEN Scanner

Star-Delta Transformation



There are some networks in which resistors are neither connected in series nor in parallel such as star-delta networks.

In such simulations, it is not possible to simplify the network by series and parallel circuit nodes however such nebwork can be simplify by using star-delta transformation technique.



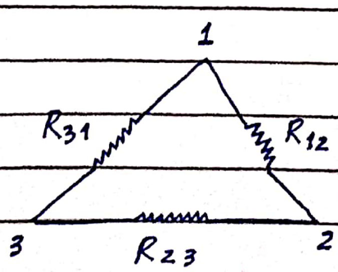
→ Delta to star transformation.

Consider three resistors R_{12}, R_{23} and R_{31} connected in Δ, these three given resistors can be replaced by three resistors R_1, R_2 and R_3 connected in star.

First we take delta connection between terminal 1 and 2, there are two parallel path one having resistance R_{12} and other having resistance $R_{23}+R_{31}$. Therefore, the equivalent resistance between one and two:

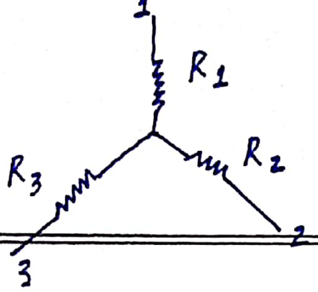
$$R_{12} \parallel (R_{31}+R_{23})$$

$$\therefore \quad R_{eq} = \frac{R_{12}(R_{31}+R_{23})}{R_{12}+R_{31}+R_{23}}$$



©Misbahul Hasan

Scanned with OKEN Scanner



Now, we take star connection, the resistance between same terminal is equal to $R_1 + R_2$.

Hence, as the terminal resistance had to be same i.e resistance between 1 and 2 for star is equal to 1 and 2 for delta.

$$\Rightarrow \quad R_1 + R_2 = \frac{R_{12}(R_{23} + R_{31})}{R_{12} + R_{23} + R_{31}} \quad \text{--- (1)}$$

Similarly,

$$R_2 + R_3 = \frac{R_{23}(R_{31} + R_{12})}{R_{12} + R_{23} + R_{31}} \quad \text{--- (2)}$$

and $$R_1 + R_3 = \frac{R_{31}(R_{12} + R_{23})}{R_{12} + R_{23} + R_{31}} \quad \text{--- (3)}$$

Now, subtracting (2) from (1)

$$\therefore \quad R_1 - R_3 = \frac{R_{12}R_{23} + R_{12}R_{31} - R_{23}R_{31} - R_{23}R_{12}}{R_{12} + R_{23} + R_{31}}$$

$$R_1 - R_3 = \frac{R_{12}R_{31} - R_{23}R_{31}}{R_{12} + R_{23} + R_{31}}$$

Adding the above result in eq (3)

$$\therefore \quad 2R_1 = \frac{R_{12}R_{31} + R_{12}R_{31}}{R_{12} + R_{23} + R_{31}}$$

$$\therefore \quad R_1 = \frac{R_{12}R_{31}}{R_{12} + R_{23} + R_{31}} \quad \text{--- (4)}$$

Similarly: $R_2 = \dfrac{R_{12} \cdot R_{23}}{R_{12} + R_{23} + R_{31}}$ (5) and $R_3 = \dfrac{R_{31} \cdot R_{23}}{R_{12} + R_{23} + R_{31}}$ (6)

©Misbahul Hasan

Scanned with OKEN Scanner

=> Star to Delta transformation

Dividing eq (4) and (5)

$$\therefore \frac{R_1}{R_2} = \frac{R_{12} \cdot R_{31}}{R_{12} \cdot R_{23}}$$

$$\frac{R_1}{R_2} = \frac{R_{31}}{R_{23}}$$

$$\therefore R_{31} = \frac{R_1 \times R_{23}}{R_2}$$

and Dividing eq (4) and (6)

$$\frac{R_1}{R_3} = \frac{R_{12}}{R_{23}}$$

$$\therefore R_{12} = \frac{R_1 \cdot R_{23}}{R_3}$$

Substituting the values of R_{12} and R_{31} in (4)

$$R_1 = \frac{\frac{R_1 \cdot R_{23}}{R_3} \times \frac{R_1 \cdot R_{23}}{R_2}}{(R_{12} + R_{23} + R_{31})}$$

$$\therefore R_{23} = R_2 + R_3 + \frac{R_2 R_3}{R_1}$$

$$R_{12} = R_1 + R_2 + \frac{R_1 R_2}{R_3}$$

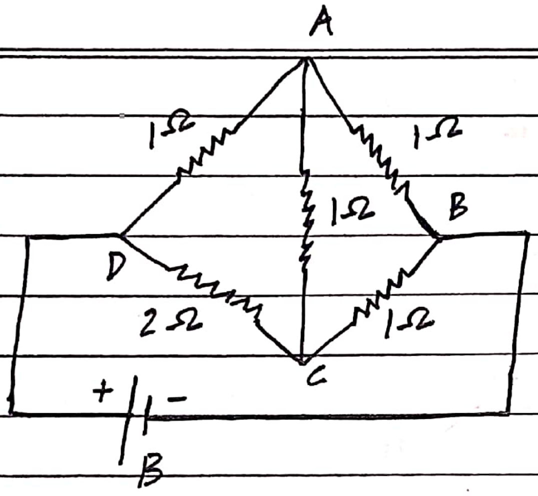
$$R_{31} = R_3 + R_1 + \frac{R_1 R_3}{R_2}$$

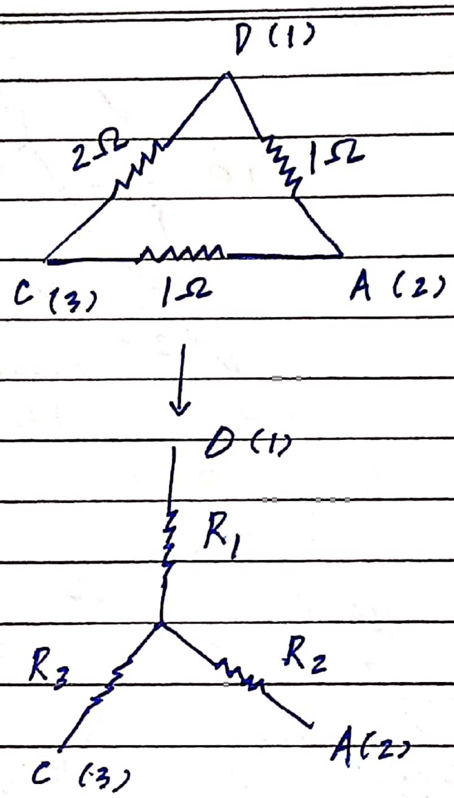
Ans.

©Misbahul Hasan

Scanned with OKEN Scanner

Q. Find the equivalent resistance across the battery terminal of the given network using Star to Delta and Delta to Star transformation technique.



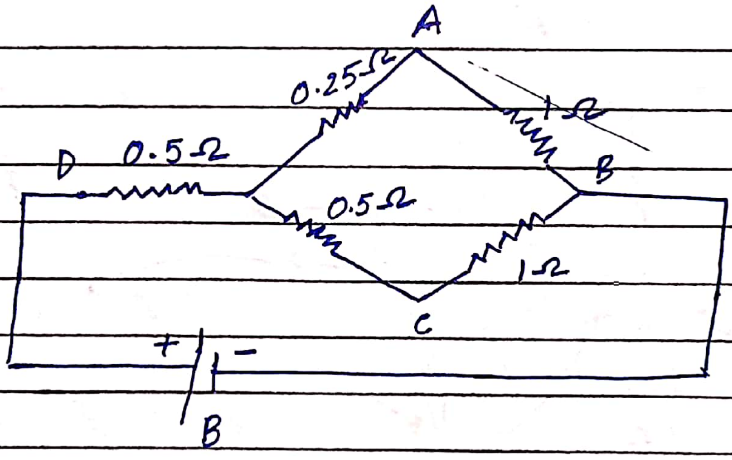


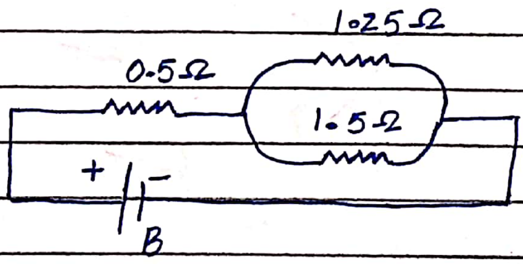
$\therefore$ $R_1 = \dfrac{1 \times 2}{4} = \dfrac{1}{2} = 0.5\ \Omega$

$R_2 = \dfrac{1 \times 1}{4} = 0.25\ \Omega$

$R_3 = \dfrac{1 \times 2}{4} = 0.5\ \Omega$

$\therefore$

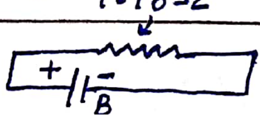




$\therefore$ $R_{eq} = 0.5 + (1.25 \,//\, 1.5)$

$R_{eq} = 1.18\ \Omega$

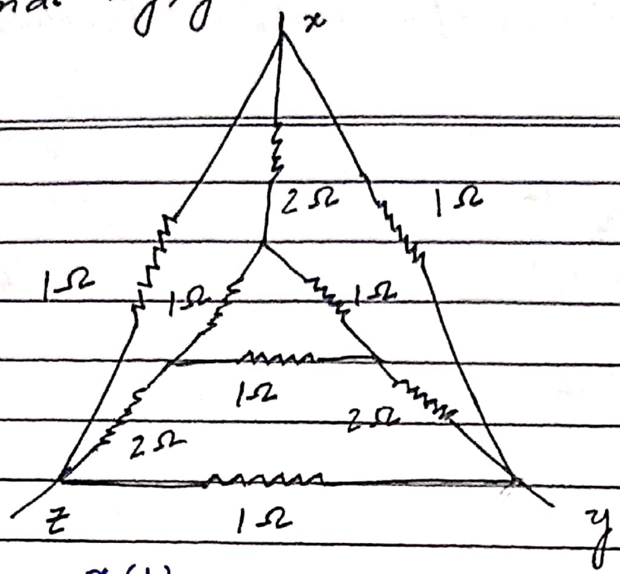
$\therefore$



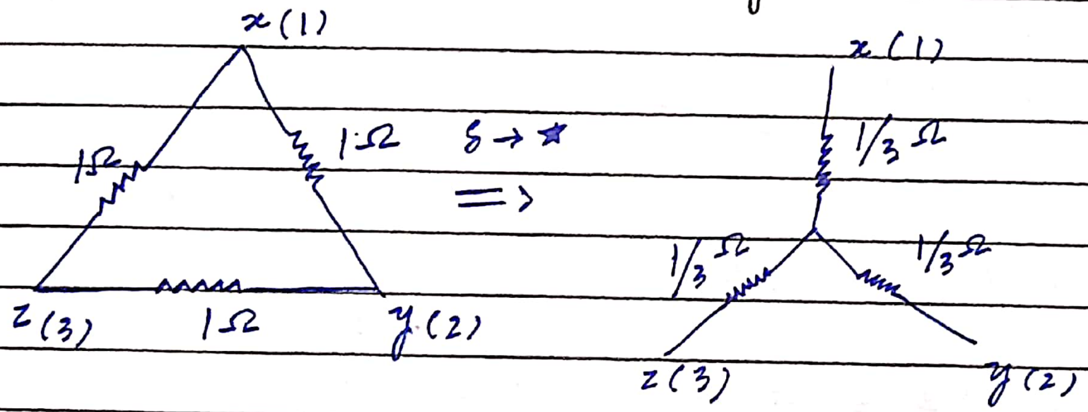
©Misbahul Hasan

Scanned with OKEN Scanner

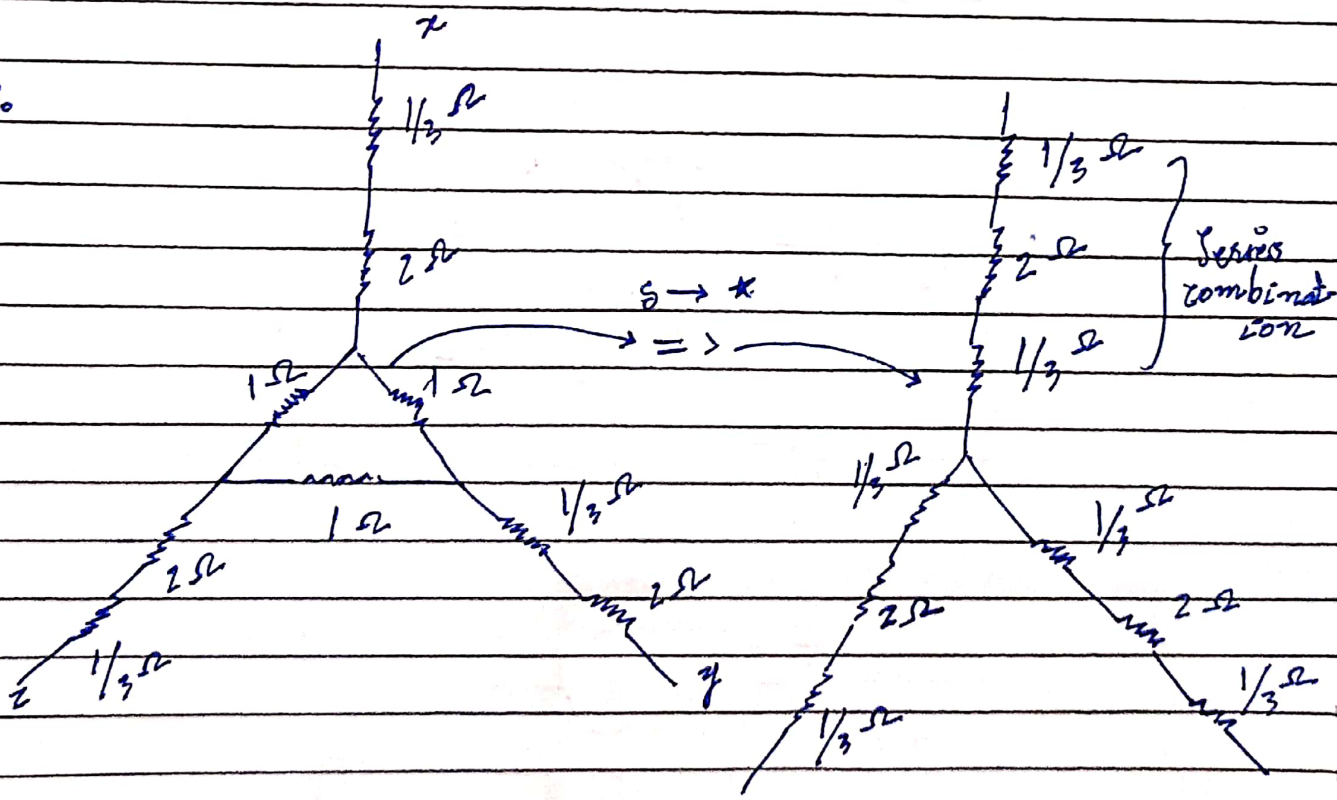
Q. Determine the equivalent resistance between the terminal xy, yz and zx.



∴



∴

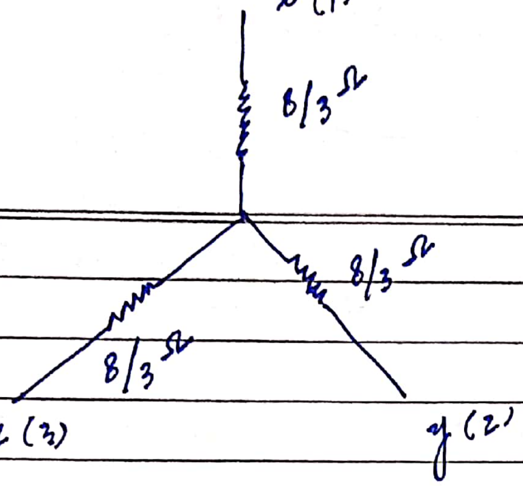


©Misbahul Hasan

Scanned with OKEN Scanner

$\frac{2}{3} + 2$

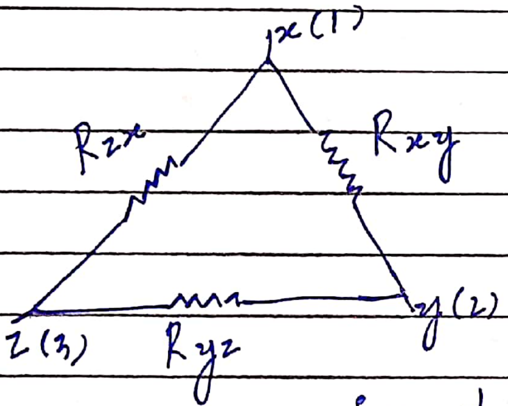
∴



اپنے ظالم کو بیچ سر پہ

$\star \to \delta$

Delta transformation



$$\therefore R_{xy} = R_x + R_y + \frac{R_x R_y}{R_z}$$

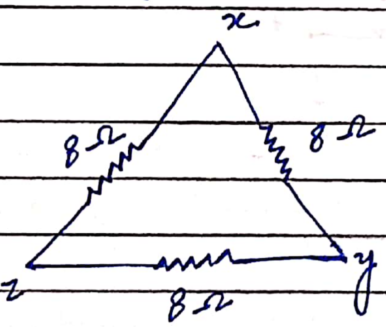
$$= \frac{8}{3} + \frac{8}{3} + \frac{(8/3)(8/3)}{(8/3)}$$

$$R_{xy} = \frac{8}{3} \times 3 = 8\,\Omega$$

Similarly; $R_{yz} = 8\,\Omega$

$R_{zx} = 8\,\Omega$

∴ The given circuit transforms into:



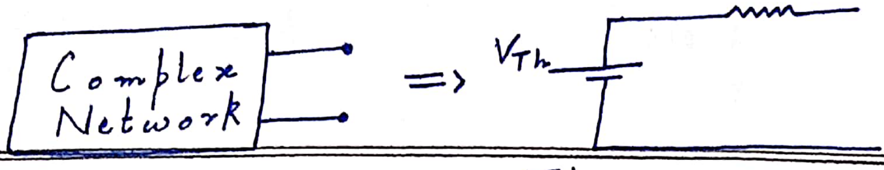
Hence,
the equivalent resistance between terminals xy, yz and zx is 8 Ω.

Ans.

©Misbahul Hasan

Scanned with OKEN Scanner

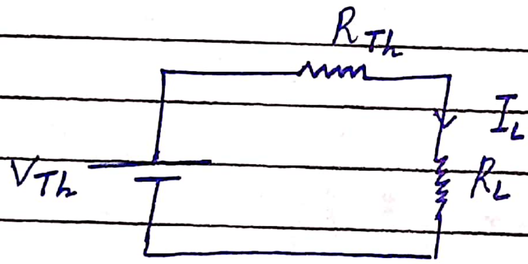
Thevenin's Theorem



Thevenin's Equivalent Network.

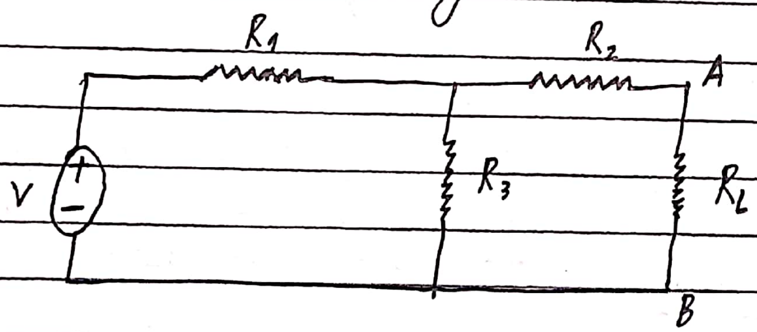
Thevenin's Theorem is the most important theorem that can be used for simplification of complicated network.

Any linear bilateral network having two terminals can be replaced by an equivalent network or circuit. Consisting a single voltage source V_{Th} in series with single resistance R_{Th}.



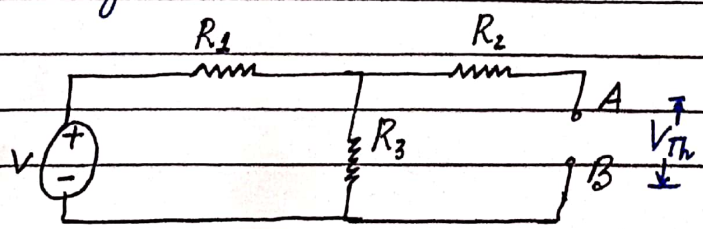
$$I_L = \frac{V_{Th}}{R_{Th} + R_L}$$

→ How to Thevenize a given network



let us consider a complicated network, we are to find current flowing through R_L by using Thevenin's Theorem.

Step 1: Remove the load resistor and calculate the Thevenin's equivalent voltage V_{Th} across AB

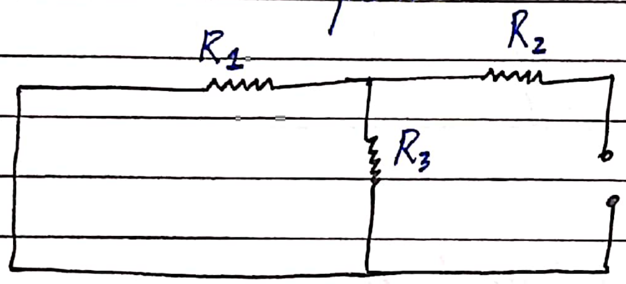


©Misbahul Hasan
Scanned with OKEN Scanner

$\therefore$ $V_{Th} = IR_3$ = voltage across R_3

$$V_{Th} = \frac{V}{R_1 + R_3} \cdot R_3$$

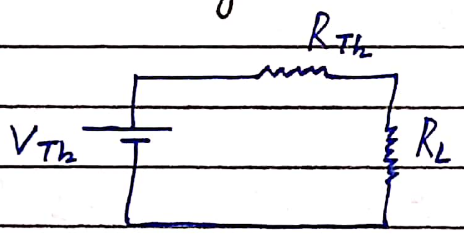
Step 2: Calculate R_{th} by voltage source shorted and current source open.



$$R_{Th} = (R_1 \parallel R_3) + R_2$$

$$R_{Th} = \frac{R_1 R_3}{R_1 + R_3} + R_2 \ \Omega.$$

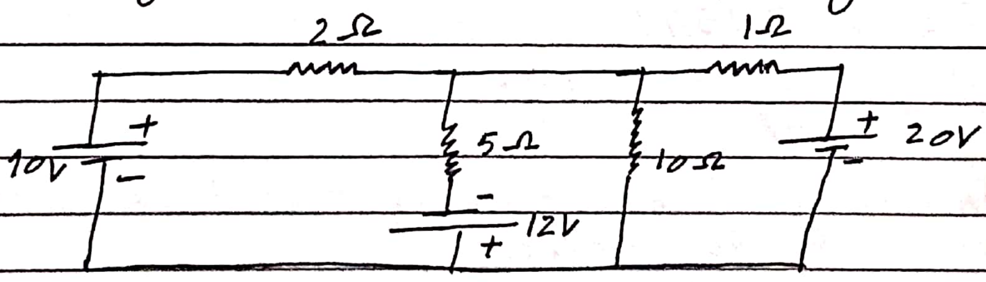
Step 3: Draw the Thevenin's equivalent network by connecting V_{Th} and R_{Th} in series.



$$\boxed{I_L = \frac{V_{Th}}{R_{Th} + R_L}}$$

NOTE: Now R_L is connected back across the terminal AB and find the load current I_L.

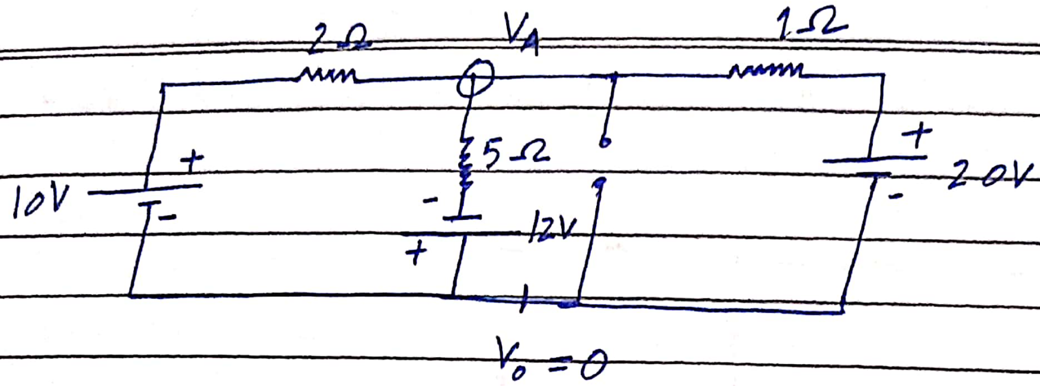
Q. Find the current through 10Ω resistance utilizing Thevenin's theorem of the given network.



©Misbahul Hasan

Scanned with OKEN Scanner

Removing $R_L = 10\Omega$ from the given circuit and calculating V_{Th}.



$\therefore$ Applying KCL at node A

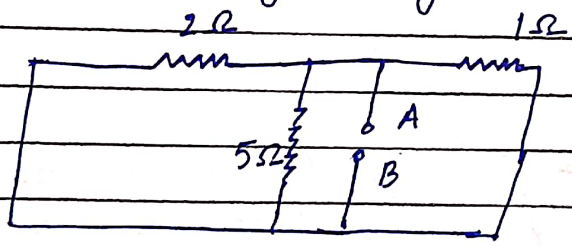
$$\frac{V_A - 10}{2} + \frac{V_A + 12}{5} + \frac{V_A - 20}{1} = 0$$

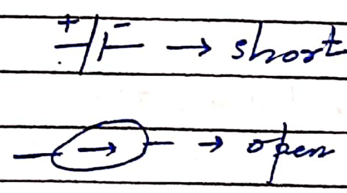
$$5V_A - 50 + 2V_A + 24 + 10V_A - 200 = 0$$

$$17V_A = 226$$

$$V_A = \frac{226}{17} = 13.29\,V$$

→ Now, shorting voltage sources and calculating R_{Th}

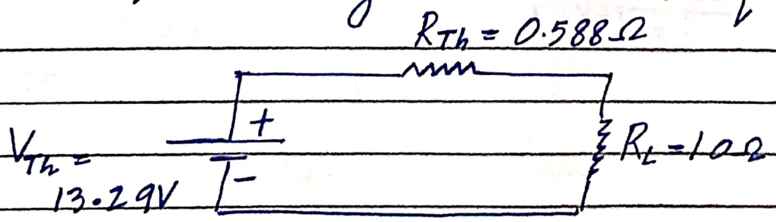




$\therefore$ $R_{Th} = 2 \| 5 \| 1$

$$R_{Th} = \frac{10}{17} = 0.588\,\Omega$$

Now, drawing Thevenin's equivalent circuit:

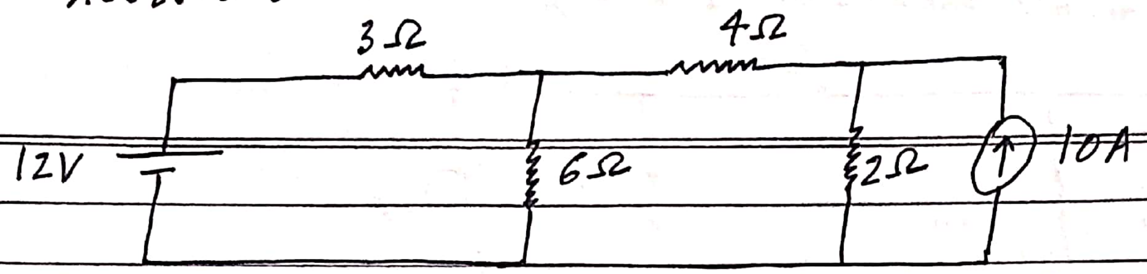


$$\therefore I_L = \frac{13.29}{0.588 + 10} = \frac{13.29}{10.588} = 1.25\,A \quad \text{Ans.}$$

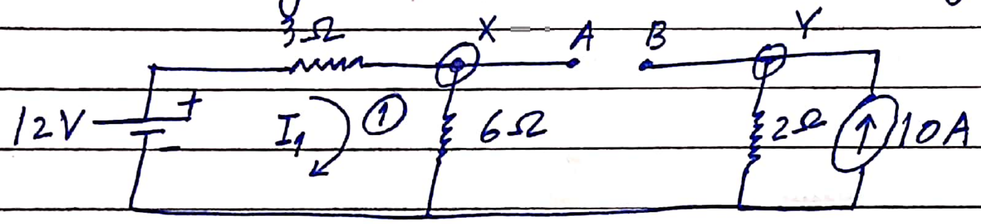
©Misbahul Hasan

Scanned with OKEN Scanner

Q. Calculate the current flowing through 4Ω resistor.



Removing $R_L = 4\Omega$ and calculating V_{Th}.



$$\therefore \quad I_1 = \frac{12}{3+6} = \frac{12}{9} = \frac{4}{3} \text{ A}$$

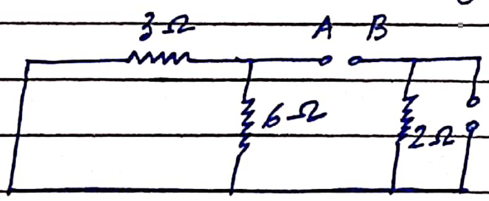
$\therefore$ Voltage dropped across $3\Omega = 3 \times 4/3 = 4V$

Hence, voltage on node $X = 12 - 4 = 8V$
and voltage on node $Y = 10 \times 2 = 20V$

$\Rightarrow \quad V_{AB} = V_{XY} = 20 - 8 = 12V = V_{Th}$.

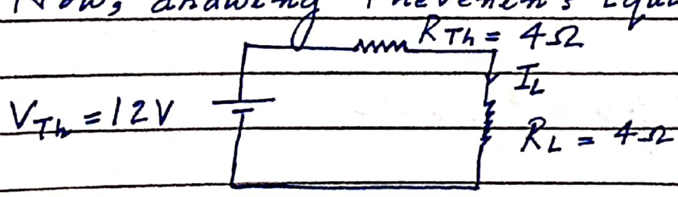
Now, shorting voltage sources and opening current sources and calculating R_{Th}.

$\Rightarrow$



$$\therefore \quad R_{Th} = (3 \| 6) + 2 = 4\Omega$$

Now, drawing Thevenin's Equivalent circuit:

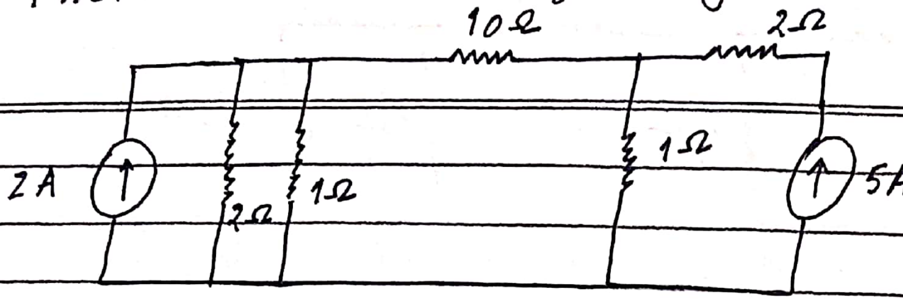


Hence,
$$I_L = \frac{12}{4+4} = 1.5 \text{ Ans.}$$

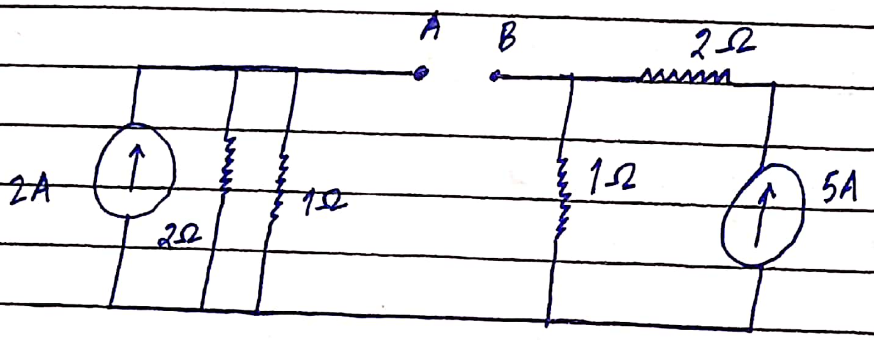
©Misbahul Hasan

Scanned with OKEN Scanner

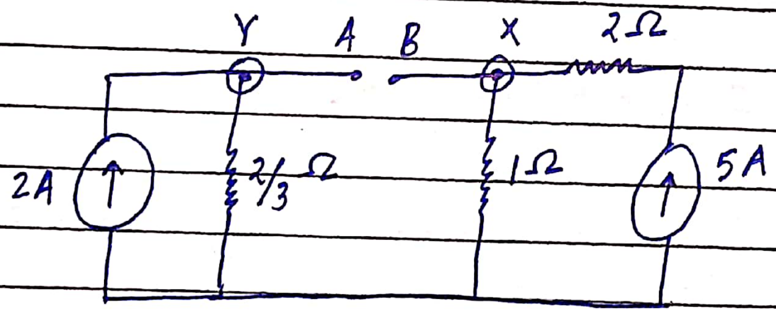
Q. Find the power loss in 10Ω resistor utilizing Thevenin's theorem of the given circuit.



Removing $R_L = 10\Omega$ and calculating V_{Th}.



∴ Replacing 2Ω and 1Ω with their equivalent resistance of $\frac{2}{3}\Omega$ (Parallel combination).



∴ Voltage on $Y = 2 \times \frac{2}{3} = \frac{4}{3} V$

∴ Voltage across $2\Omega = 5 \times 2 = 10V$

Voltage across $1\Omega = 5 \times 1 = 5V$

∴ Voltage on $X = 10 - 5 = 5V$

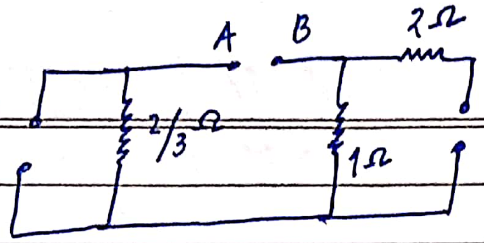
Now,

$V_{Th} = V_{AB} = 5 - \frac{4}{3} = \frac{11}{3} V = 3.66 V$.

©Misbahul Hasan

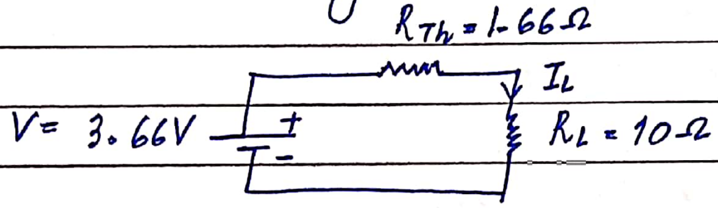
Scanned with OKEN Scanner

Now, shorting voltage sources and opening current sources, and calculating R_{Th}.



$$\therefore \quad R_{Th} = 1 + \frac{2}{3} = \frac{5}{3} = 1.66\,\Omega.$$

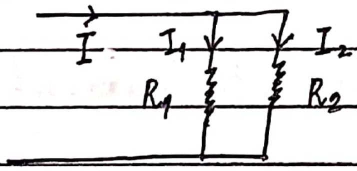
Now, drawing Thevenin's equivalent circuit.



$$\therefore \quad I_L = \frac{3.66}{1.66+10} = \frac{3.66}{11.66} = 0.31\,A$$

$\Rightarrow$ Power dissipated in $10\Omega = I^2R = (0.31)^2 \times 10$
$= 0.961\,W$ Ans.

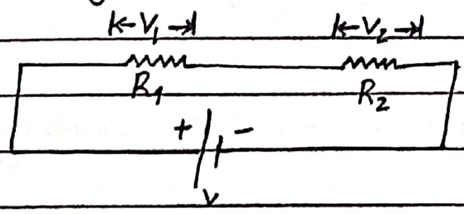
→ Current Divider Rule



$$I_1 = \frac{I \cdot R_2}{R_1 + R_2}$$

$$I_2 = \frac{I \cdot R_1}{R_1 + R_2}$$

→ Voltage Divider Rule



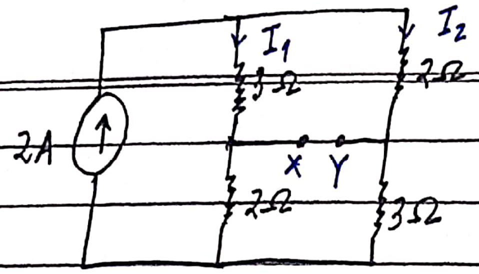
$$V_1 = \frac{R_1 \cdot V}{R_1 + R_2}$$

$$V_2 = \frac{R_2 \cdot V}{R_1 + R_2}$$

©Misbahul Hasan

Scanned with OKEN Scanner

Q. Find the Thevenin's equivalent network across the terminal xy in the given terminal.



$I_1 = 1A$
$I_2 = 1A$

[Because both arms have same resistance, hence current divides equally]

$\Rightarrow \quad V_X = 3 \times I_1 = 3\,V$

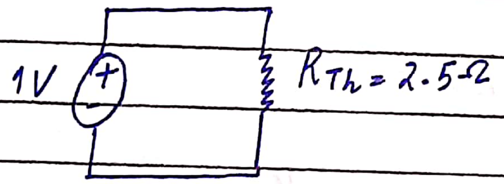
$V_Y = 2 \times I_2 = 2\,V$

$\therefore \quad V_{XY} = 3 - 2 = 1V = V_{Th}$

and $R_{Th} = (3 \parallel 2) + (3 \parallel 2)$

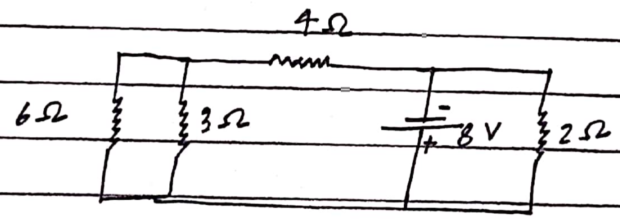
$= \frac{6}{5} + \frac{6}{5} = \frac{12}{5} = 2.5\Omega$

$\therefore$ Drawing Thevenin's equivalent network

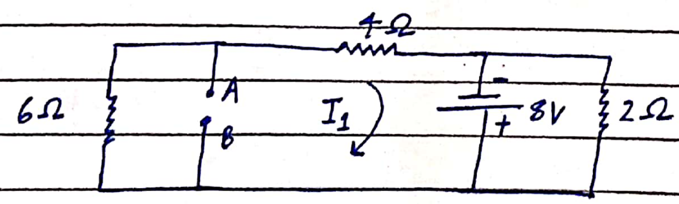


Ans.

Q. Using Thevenin's Theorem, calculate the current through 3Ω resistor.



Removing the load resistor $R_L = 3\Omega$ and calculating V_{Th}.



©Misbahul Hasan
Scanned with OKEN Scanner

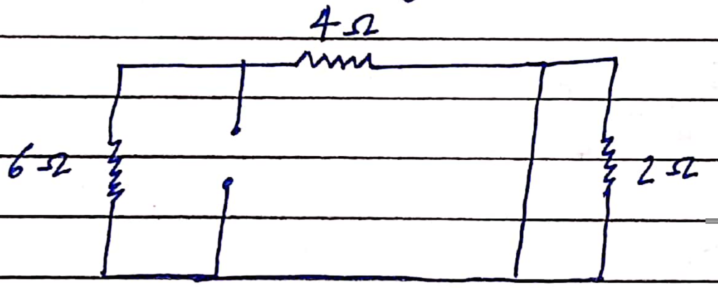
∴ Calculating I_1 from mesh 1

$$10 I_1 = 8$$
$$I_1 = 0.8A$$

∴ Voltage across $6\Omega = 6 \times 0.8 = 4.8V$

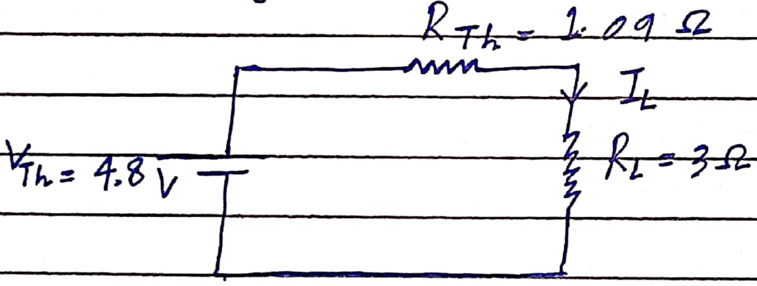
This is equal to the voltage across AB i.e V_{th} as AB is parallel to 6Ω.

Now, calculating R_{Th}



$$\therefore R_{Th} = (6 \parallel 4) \parallel 2$$
$$= (2.4\Omega) \parallel (2\Omega)$$
$$= 1.09\Omega$$

Drawing Thevenin's Equivalent circuit :

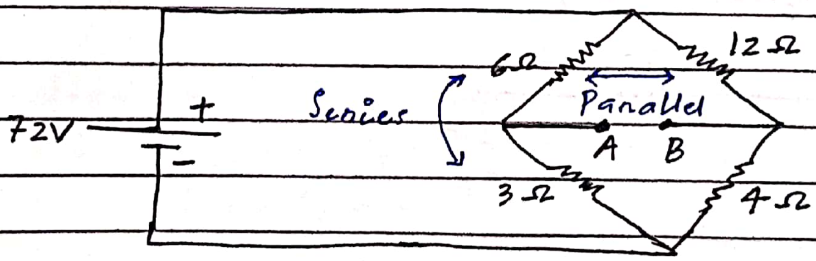


$$\Rightarrow I_L = \frac{4.8}{1.09+3} = 1.173A$$

Ans.

इस सवाल मे गड़बड़ हो सकती है। बेहतर है के आप देख ले ~~एक~~ एक बार।

Q. Find the Thevenin's equivalent network across the terminal AB after given network ;



©Misbahul Hasan

Scanned with OKEN Scanner

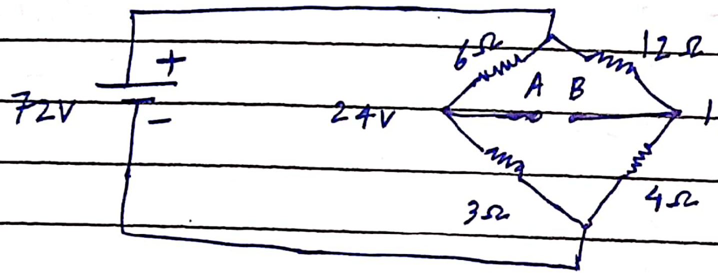
Applying voltage divider for 6Ω resistor

$\therefore\ V_{6\Omega} = \frac{6 \times 72}{6+3} = 48V$ (6 and 3Ω resistors are in series)

Similarly for 12Ω resistor

$\therefore\ V_{12} = \frac{12 \times 72}{12+4} = 54V$ (12 and 4Ω resistors are in series).

Now;

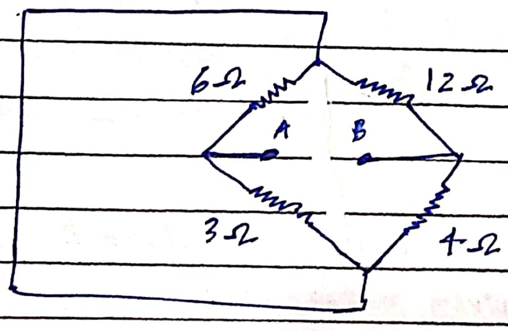


$\therefore\ V_A = 72 - 48 = 24V$

$V_B = 72 - 54 = 18V$

$\therefore\ V_{Th} = V_A - V_B = 24 - 18 = 6V$

Now, calculating R_{Th}

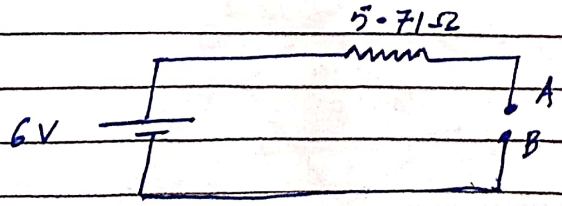


$R_{Th} = (6 \| 12) + (3 \| 4)$

$= 4 + 12/7$

$R_{Th} = 40/7\ \Omega = 5.71\Omega$

$\therefore$ Thevenin's Equivalent Network;

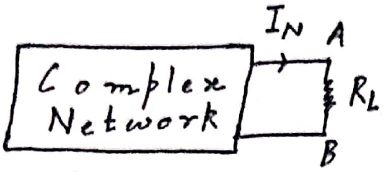


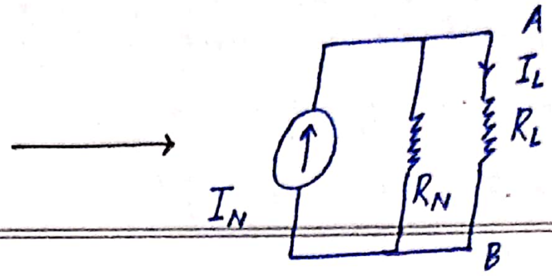
Ans.

©Misbahul Hasan

Scanned with OKEN Scanner

Norton's Theorem





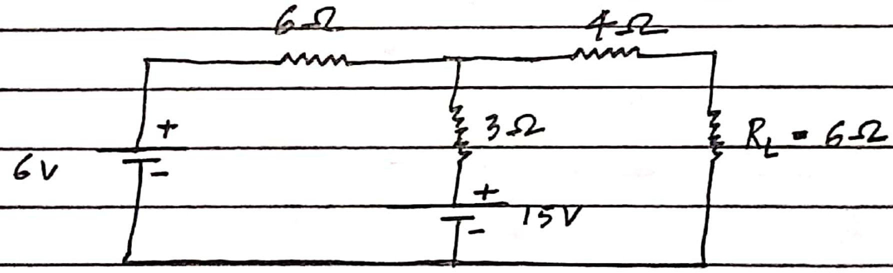
Norton equivalent circuit.

$$I_L = \frac{R_N I_N}{R_N + R_L}$$

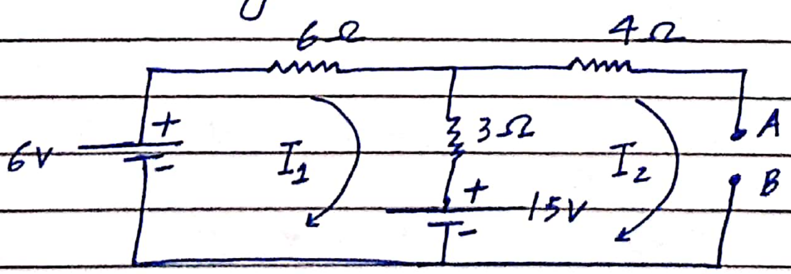
→ Norton Theorem is an alternative theorem to the Thevenin's theorem and applicable for a complex network. By using this theorem we can replace a complex network by a current in parallel with a single resistor.

Any linear bilateral network having a terminal A and B can be replaced by single current source I_N in parallel with a single resistor R_L.

Q. Find the current across $R_L = 6\Omega$, using Norton's Theorem.



Removing $R_L = 6\Omega$ and calculating I_N



Applying KVL in mesh (1)

$$+6 - 6I_1 - 3(I_1 - I_2) - 15 = 0$$

$$-9I_1 + 3I_2 = 9 \quad \text{——} (1)$$

©Misbahul Hasan
Scanned with OKEN Scanner

Applying KVL in mesh 2

$$15 - 3(I_2 - I_1) - 4I_2 = 0$$

$$-7I_2 + 3I_1 = -15 \quad \text{——} \quad (2)$$

Solving 1 and 2

$$-9I_1 + 3I_2 = 9$$

$$9I_1 - 2I_2 = -45$$

$$+ \quad + \quad +$$

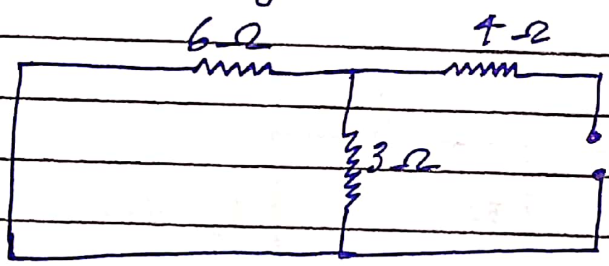
$$-18I_2 = -36$$

$$I_2 = \frac{36}{18} = 2A$$

And;

$$I_2 = I_N = 2A$$

Now, calculating R_N;

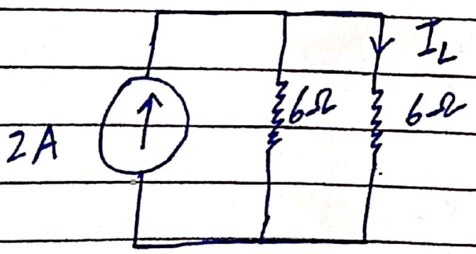


$$\therefore \quad R_N = (6 \| 3) + 4$$

$$= \frac{18}{9} + 4$$

$$R_N = 6\Omega \text{ Ans.}$$

Drawing, Norton's equivalent circuit:

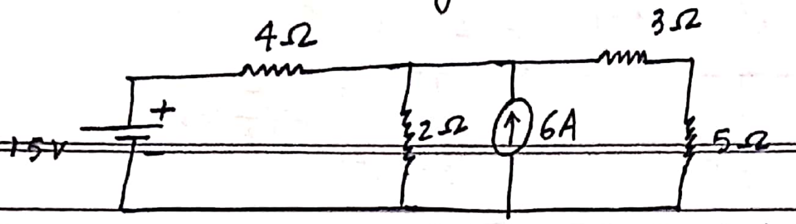


$$I_L = \frac{I_N R_N}{R_L + R_N}$$

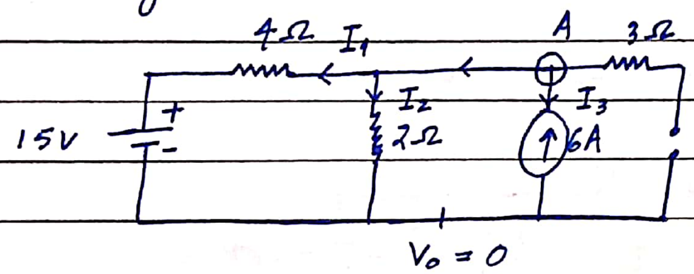
$$I_L = \frac{6 \times 2}{12} = 1A \text{ ans.}$$

©Misbahul Hasan
Scanned with OKEN Scanner

Q. Determine the current $5\,\Omega$ resistor using Norton Theorem in given network.



Removing $R_L = 5\,\Omega$ and calculating I_N



Applying Nodal Analysis at 'A'

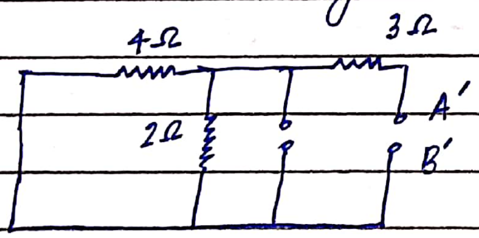
∴ Assuming I_1, I_2 and I_3 as outgoing current from node A.

=> Applying KCL.

$$\therefore \frac{V_A - 15}{4} + \frac{V_A}{2} - 6 = 0$$

$$\therefore \boxed{V_A = 13V} = V_N$$

Now, calculating R_N

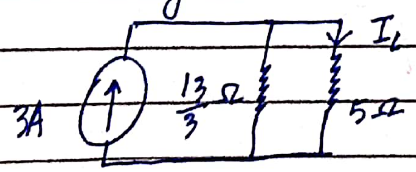


$$R_N = (4 // 2) + 3$$

$$R_N = 4/3 + 3 = \frac{13}{3}\,\Omega$$

$$\therefore I_N = \frac{V_N}{R_N} = \frac{13 \times 3}{13} = 3\,A$$

Now, drawing Norton's equivalent circuit;



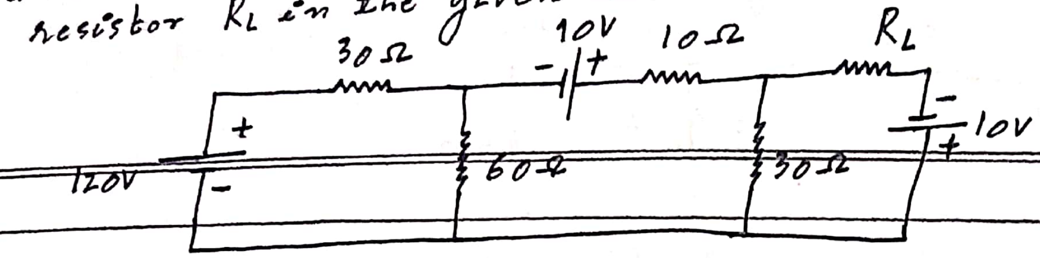
$$\Rightarrow I_L = \frac{R_N I_N}{R_N + R_L} = \frac{\frac{13}{3} \times 3}{\frac{13}{3} + 5} = 1.39\,A$$

Ans.

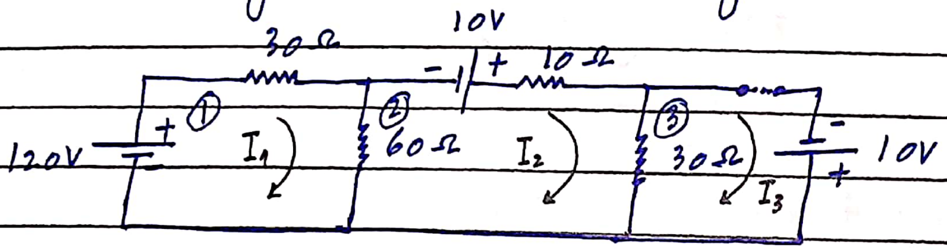
©Misbahul Hasan

Scanned with OKEN Scanner

Q. Find the Norton equivalent network across the resistor R_L in the given network.



Removing R_L and calculating I_N ;



$\therefore$ KVL for mesh ①

$$120 - 30I_1 - 60I_1 + 60I_2 = 0$$

$$-90I_1 + 60I_2 = -120$$

$$-3I_1 + 2I_2 = -4 \quad \text{——} \quad (1)$$

KVL for mesh ②

$$-60I_2 + 60I_1 + 10 - 10I_2 - 30I_2 + 30I_3 = 0$$

$$-100I_2 + 60I_1 + 30I_3 = -10$$

$$6I_1 - 10I_2 + 3I_3 = -1 \quad \text{——} \quad (2)$$

KVL for mesh ③

$$-30I_3 + 30I_2 + 10 = 0$$

$$3I_2 - 3I_3 = -1 \quad \text{——} \quad (3)$$

Arranging mesh equations into matrix form and using cramer's rule.

$$\begin{bmatrix} -3 & 2 & 0 \\ 6 & -10 & 3 \\ 0 & 3 & -3 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} -4 \\ -1 \\ -1 \end{bmatrix}$$

$$\therefore \Delta = -3(30-9) - 2(-18)$$

$$= -3(21) + 36$$

$$= -63 + 36$$

$$\Delta = -27$$

©Misbahul Hasan

Scanned with OKEN Scanner

$$\Delta_3 = \begin{vmatrix} -3 & 2 & -4 \\ 6 & -10 & -1 \\ 0 & 3 & -1 \end{vmatrix}$$

$$= -3(10+3) - 2(-6) - 4(18)$$

$$= -39 + 12 - 72$$

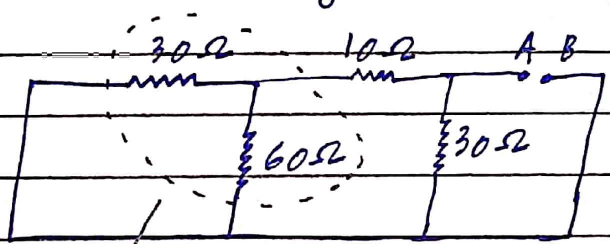
$$\Delta_3 = -99$$

$$\therefore \quad I_3 = \frac{\Delta_3}{\Delta} = \frac{-99}{-27} = \frac{11}{3} = 3.66\,A$$

Ans.

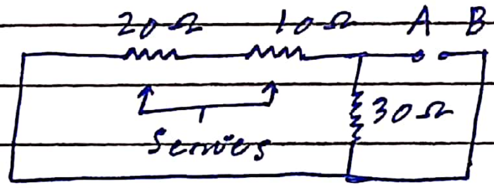
$$\Rightarrow \quad I_N = 3.66\,A$$

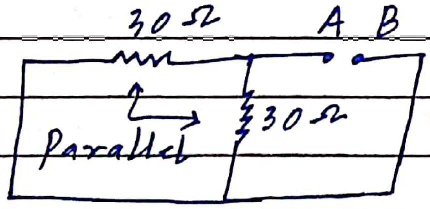
Now, calculating R_{Th}



Parallel combination

$R_{eq} = 20\,\Omega$

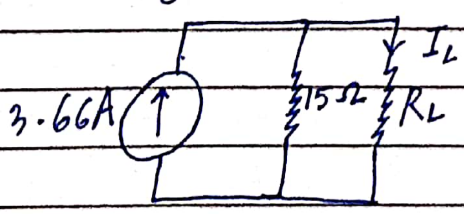




$$\Rightarrow \quad R_N = 15\,\Omega$$

$$R_{eq} = \frac{30 \times 30}{60} = 15\,\Omega$$

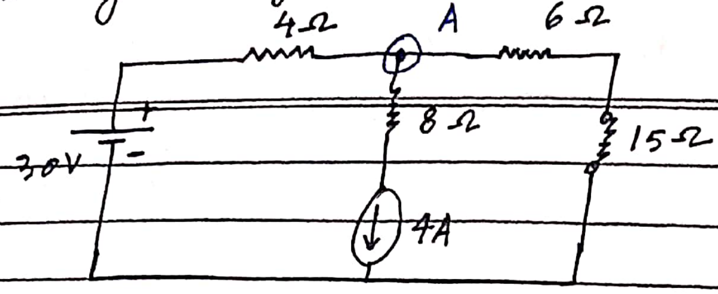
Now, drawing Norton's equivalent Network:



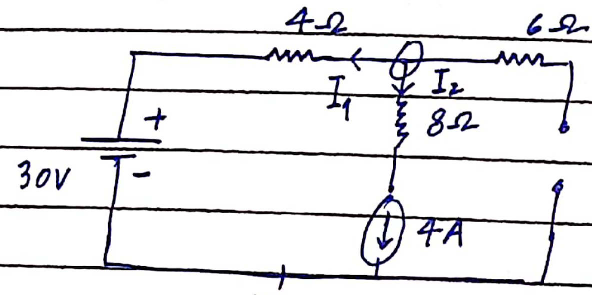
©Misbahul Hasan

Scanned with OKEN Scanner

Q. Using Norton Theorem, evaluate the current flowing through 15 Ω resistor.



Removing $R_L = 15\,\Omega$ and applying KCL at 'A'

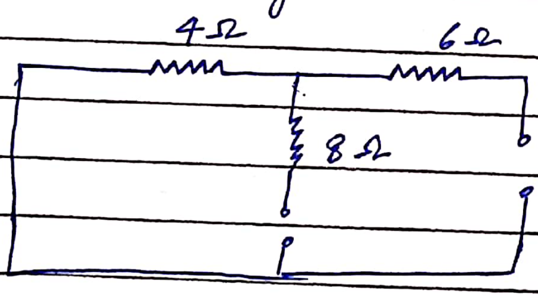


$$\therefore \quad \frac{V_A - 30}{4} + 4 = 0$$

$$V_A - 30 = -16$$

$$V_A = 14V = V_{Th}$$

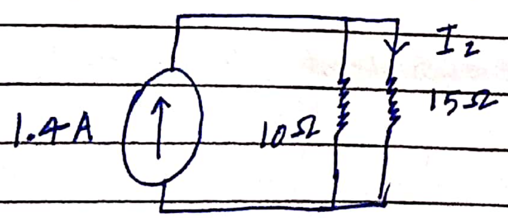
Now, calculating R_N:



$R_N = 4 + 6 = 10\,\Omega$

$\Rightarrow I_N = \dfrac{V_N}{R_N} = \dfrac{14}{10} = 1.4A$

Drawing: Norton's equivalent circuit ⇒



$\therefore I_2 = \dfrac{1.4 \times 10}{10 + 15} = \dfrac{14}{25} = 0.55A$

Ans.

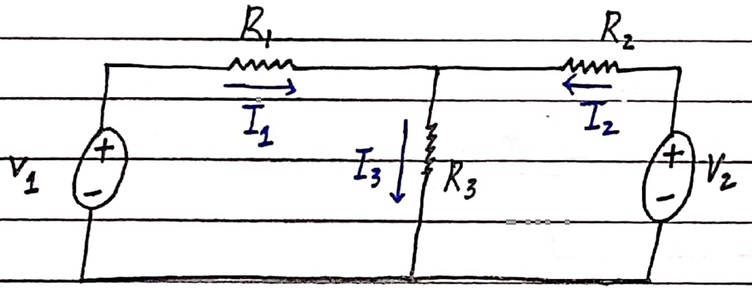
©Misbahul Hasan

Scanned with OKEN Scanner

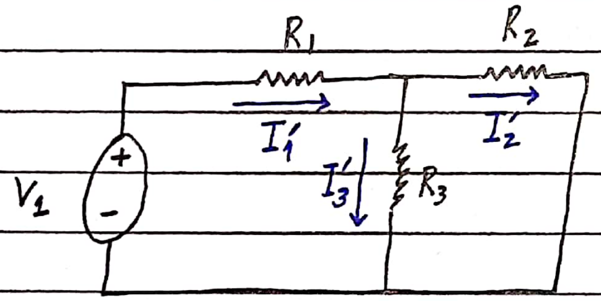
Superimposition Theorem

This theorem finds use in solving a network where two or more sources are present and not connected in series or parallel.

In a linear bilateral network containing several energy sources, the overall current in any branch is equal to the algebraic sum of current produced by each sources acting alone while the other sources are inactive.



Case 1: When $V_1 \neq 0$, $V_2 = 0$ (inactive)



Consider a complicated network having two voltage sources V_1 and V_2. Let us calculate the current in each branch using superposition

According to superposition theorem, select a single source in the given network that means $V_1 \neq 0$ and $V_2 = 0$ (short circuited).

$$R_{eq} = \frac{R_2 \cdot R_3}{R_2 + R_3} + R_1$$

$$\therefore I_1' = \frac{V_1}{\left(\frac{R_2 \cdot R_3}{R_2 + R_3} + R_1\right)}$$

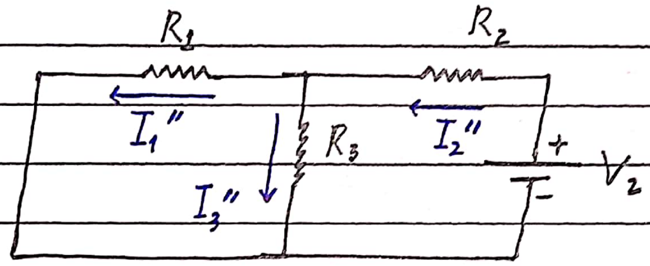
©Misbahul Hasan
Scanned with OKEN Scanner

and,
using current divider rule,

$$I_2' = \frac{I_1' R_3}{R_2 + R_3}.$$

$$I_3' = \frac{I_1' R_2}{R_2 + R_3}.$$

Case 2: When $V_2 \neq 0$, $V_1 = 0$ (inactive)



$$R_{net} = (R_1 \parallel R_3) + R_2$$
$$= \left(\frac{R_1 R_3}{R_1 + R_3} + R_2\right)$$

$$I_2'' = \frac{V_2}{R_{net}}$$

and $$I_1'' = \frac{R_3}{R_1 + R_3} \times I_2''$$

$$I_3'' = \frac{R_1}{R_1 + R_3} \times I_2''$$

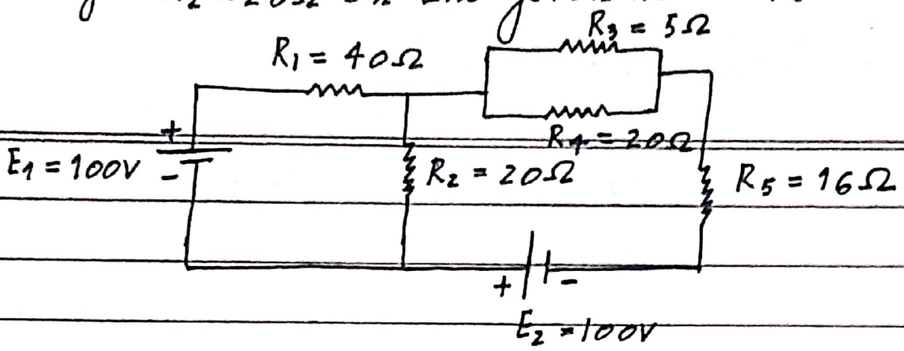
or $$I_3'' = I_2'' - I_1''$$

$\therefore$
$$I_1 = I_1' - I_1''$$
$$I_2 = I_2' - I_2''$$
$$I_3 = I_3' - I_3''.$$

©Misbahul Hasan

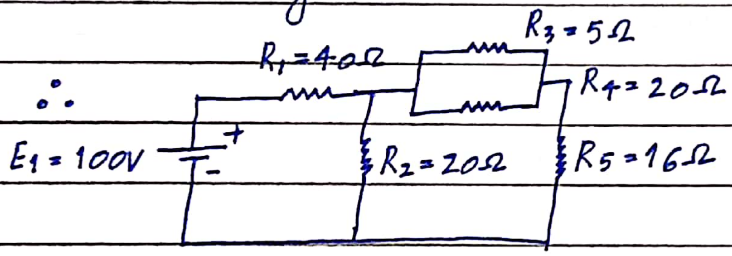
Scanned with OKEN Scanner

Example: Utilising superposition theorem, find the current through $R_2 = 20\Omega$ in the given network.



Considering E_1 as active source and rest as inactive

$\therefore$



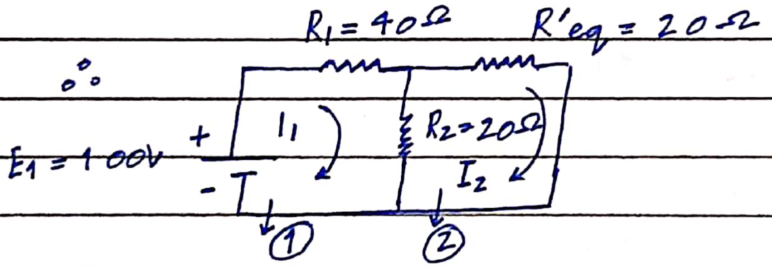
$\therefore$ R_3 and R_4 are in parallel combination

$\Rightarrow$ $R_{eq} = \frac{\cancel{5} \times \cancel{20}}{\cancel{25}} = 4\Omega$

and R_{eq} and R_5 are in series

$\Rightarrow$ $R'_{eq} = 4 + 16 = 20\Omega$

$\therefore$



$\therefore$ Applying KVL in mesh 1

$$100 - 40I_1 - 20(I_1 - I_2) = 0$$
$$-6\cancel{0}I_1 + 2\cancel{0}I_2 = -10\cancel{0}$$
$$3I_1 - I_2 = 5 \quad \text{—— (1)}$$

Applying KVL in mesh 2

$$-20I_2 - 20(I_2 - I_1) = 0$$
$$-40I_2 + 20I_1 = 0$$
$$I_1 = 2I_2 \quad \text{—— (2)}$$

$\therefore$ $6I_2 - I_2 = 5$ $\Rightarrow$ $\boxed{I_2 = 1A}$ and $\boxed{I_1 = 2A}$

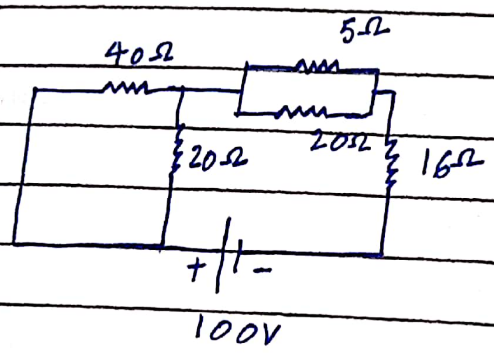
©Misbahul Hasan

Scanned with OKEN Scanner

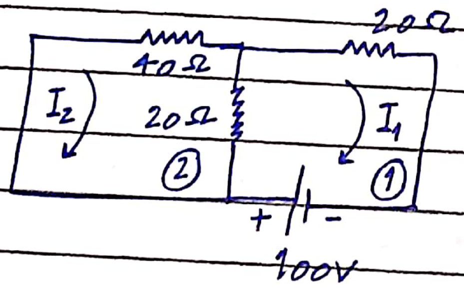
Hence,

$$I_{R_2=20\Omega} = I_1 - I_2 = 2-1 = 1A.$$

Now, considering E_2 as active source and others as inactive sources.



This network can be simplified into;



∴ Applying KVL in mesh 1

$$100 - 20(I_1 - I_2) - 20I_1 = 0$$
$$-40I_1 + 20I_2 = -100$$
$$2I_1 - I_2 = 5 \quad \text{--- (1)}$$

Applying KVL in mesh 2

$$-40I_2 - 20(I_2 - I_1) = 0$$
$$-60I_2 + 20I_1 = 0$$
$$I_1 = 3I_2 \quad \text{--- (2)}$$

∴ $6I_2 - I_2 = 5$

$\boxed{I_2 = 1A}$, $\boxed{I_1 = 3A}$

∴ $I'_{R_2=20\Omega} = I_1 - I_2 = 3-1 = 2A$

©Misbahul Hasan

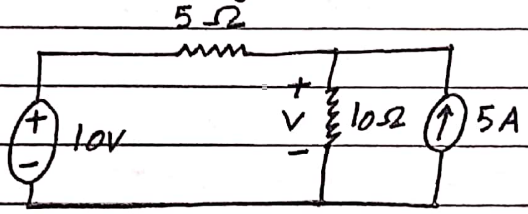
Scanned with OKEN Scanner

Current through $R_2 = 20\Omega$, when both sources are active

$$(I_{R_2=20\Omega})_{\substack{\text{Both source} \\ \cancel{\text{active}}}} = (I_{R_2=20\Omega})_{\substack{E_1=\text{active} \\ \cancel{E_2=0}}} + (I'_{R_2=20\Omega})_{E_2=\text{active}}$$

$$I_{R_2=20\Omega} = 1+2 = 3\,A \quad \underline{\underline{\text{Ans.}}}$$

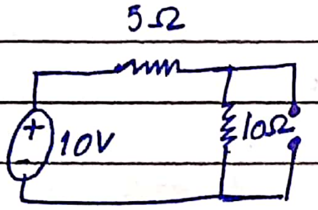
Q. Find the voltage V by using superimposition theorem in the given ~~nature~~ network.



★ دھیان دے!!

In active state: Voltage source is replaced by a short wire i.e short circuited while current source is replaced by an open wire i.e it is opened.

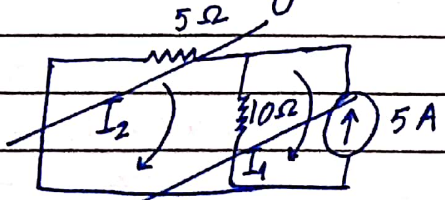
First considering 10V as an active source.



$\therefore$ $I = \dfrac{10}{15} = \dfrac{2}{3}$

$\therefore$ $V'_{10\Omega} = 10 \times \dfrac{2}{3} = 6.66\,V$

Now, considering 5A as an active source:



~~$I_1 = -5A$~~

$\therefore$ Applying KVL for ~~mesh~~ (1)

~~$-10(I_1 - I_2) = 0$~~

~~$10I_1 = 10I_2$~~

~~$I_1 = I_2$~~

~~$-5I_2 - 10(I_2 - I_1) = 0$~~

~~$-15I_2 + 10I_1 = 0$~~

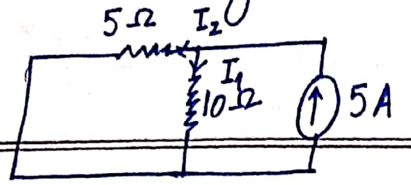
~~$\therefore \frac{3}{2}I_2 = I_1$~~

©Misbahul Hasan

Scanned with OKEN Scanner

Now, considering 5A as an active source

$\therefore$



$\therefore$ Using Current Divider rule

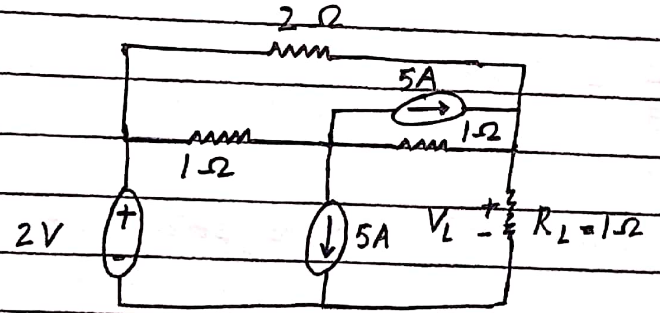
$$I_1 = \frac{\cancel{5}\times 5}{\cancel{15}\,3} = \frac{5}{3}\ A$$

$$\Rightarrow V''_{10\Omega} = \frac{10\times 5}{3} = \frac{50}{3}\ V = 16.66V$$

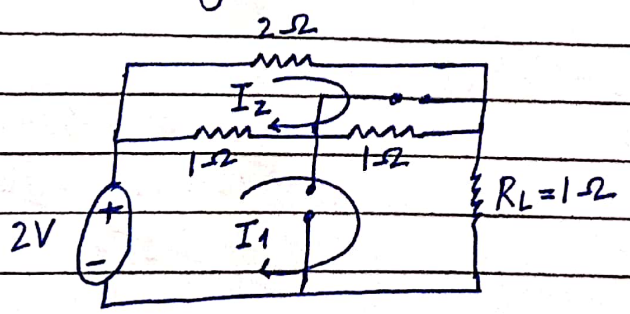
$\therefore$ Voltage across 10Ω when both sources are active ;

$$\Rightarrow V_{10\Omega} = V'_{10\Omega} + V''_{10\Omega}$$
$$= 6.66 + 16.66$$
$$V_{10\Omega} = 23.32V$$ Ans.

Q. Find the voltage V_L in the given network using superposition theorem.



Considering 2V as an active source ;



©Misbahul Hasan

Scanned with OKEN Scanner

Applying KVL in mesh 1

$-2(I_1-I_2)-I_1+2=0$

$-3I_1+2I_2=-2$ —— ①

Applying KVL in mesh 2

$-2(I_2-I_1)-2I_2=0$

$-4I_2+2I_1=0$

$4I_2=2I_1$

$\boxed{I_1=2I_2}$ —— ②

Putting ② in ①

$-6I_2+2I_2=-2$

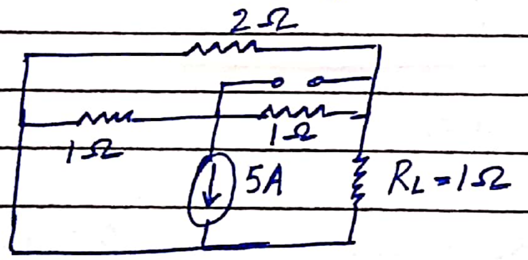
$-4I_2=-2$

$\boxed{I_2=0.5A}$

$\boxed{I_1=1A}$

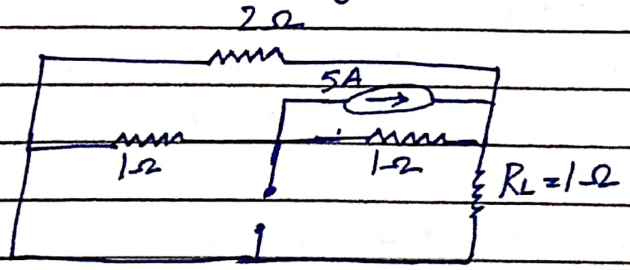
$\therefore \boxed{V_L'=1V}$

Now, considering '5A' as an active source



$\therefore \quad V_L''=5\times1=5V$

Now, considering '5A' as an active source



$\therefore \quad V_L'''=5\times1=5V$

Hence $V_L=V_L'+V_L''+V_L'''$

$V_L=1+5+5=11$ Ans.

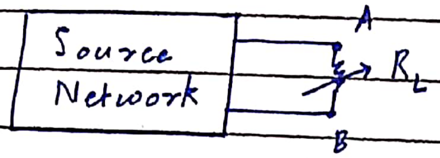
©Misbahul Hasan

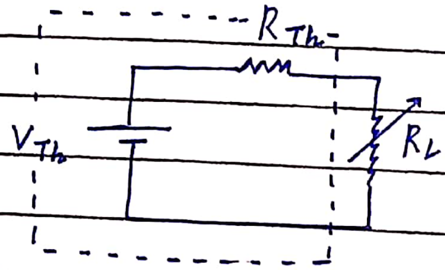
Scanned with OKEN Scanner

Maximum Power Transfer Theorem

Maximum Power Transfer Theorem deals with power from a source to load. This theorem is used to find the value of load resistance for which there would be maximum amount of power from source to load.

A load resistance, being connected to a DC network receives maximum power when the load resistance is equal to the internal resistance of the source network.





$R_L = R_{Th}$

Maximum power delivered.

$$P = I_L^2 \cdot R_L$$

$$P = \frac{V_{th}^2}{(R_{th}+R_L)^2} \cdot R_L$$

$$P = \frac{V_{th}^2}{(2R_L)^2} \cdot R_L$$

$$\boxed{P_{max} = \frac{V_{th}^2}{4R_L}}$$

This theorem states that maximum power from a complex network is obtained when the load resistance R_L is equivalent to Thevenin's equivalent resistance R_{th} as seen from the load terminals.

Consider a variable resistance R_L is connected to DC source network having V_{Th} (Thevenin's equivalent voltage) and Thevenin's equivalent resistance R_{Th} of the source network. Therefore, we know that $\frac{V_{Th}}{R_{Th}+R_L} = I_L$ and the power delivered to load resistor is

$$P = I_L^2 \times R_L$$

$$P = \left(\frac{V_{Th}}{R_{Th}+R_L}\right)^2 \times R_L$$

©Misbahul Hasan

Scanned with OKEN Scanner

Power P can be maximized by varying R_L when maximum power can be delivered to the load. When

$$P_{L(max)} = \frac{dP}{dR_L} = 0$$

$$\therefore \quad \frac{d}{dR_L}\left(\left(\frac{V_{th}}{R_{th}+R_L}\right)^2 \times R_L\right) = 0$$

$$R_L \cdot V_{th}^2 \left(\frac{d\,(R_{th}+R_L)^{-2}}{dR_L}\right) + \frac{V_{th}^2}{(R_{th}+R_L)^2} = 0$$

$$\frac{R_L \times V_{th}^2 \times (-2)}{(R_{th}+R_L)^3} + \frac{V_{th}^2}{(R_{th}+R_L)^2} = 0$$

$$\therefore \quad -2 \times R_L \times \cancel{V_{th}^2} = -\cancel{V_{th}^2} \times R_{th} - \cancel{V_{th}^2} \cdot R_L$$

$$\Rightarrow \boxed{R_L = R_{Th}}$$

Hence, $R_L = R_{Th}$, the system being perfectly matched for load and source. The power transfer becomes maximum and this amount of power is

$$P = \frac{V_{th}^2}{(R_L+R_{Th})^2} \times R_L = \frac{V_{th}^2}{4R_L^2} \times R_L = \frac{V_{th}^2}{4R_L}.$$

→ Efficiency (η): $\dfrac{\text{Output Power}}{\text{Input Power}}$

$$= \frac{I_L^2 \cdot R_L}{I_L^2\,(R_{Th}+R_L)} = \frac{R_L}{R_{th}+R_L}$$

Therefore maximum efficiency:

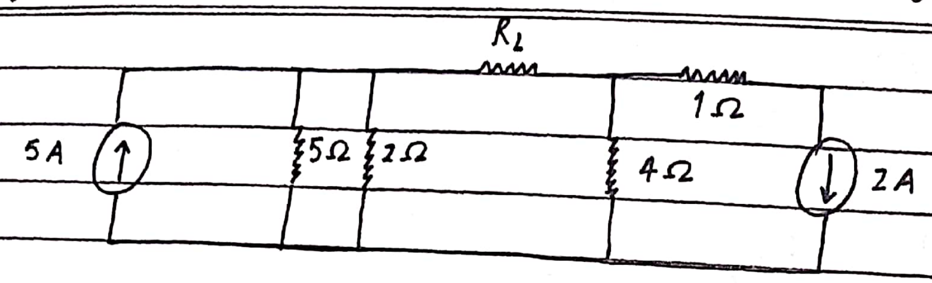
$$P = \frac{R_{Th}}{2R_{Th}} = 0.5 = 50\%$$

NOTE: Efficiency in maximum power transfer is only 50% as one half of the total power generated in internal resistance of the source network.

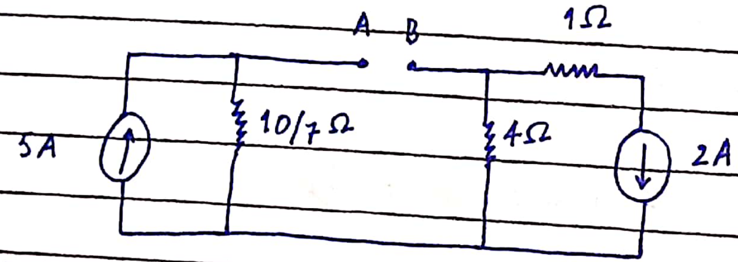
©Misbahul Hasan

Scanned with OKEN Scanner

Q. Find the value of R_L such that maximum power transfer takes place from the current sources to the load resistor R_L and also obtain the amount of maximum power transferred in the given network.



Removing load resistance and calculating V_{Th}.

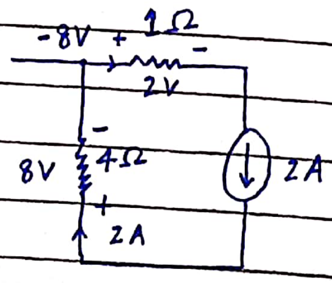


Replacing 5Ω and 2Ω with their eq. resistance.

$\therefore$ $Req = (5 \parallel 2) = 10/7\,\Omega$

$$\therefore \quad V_A = 5\left(\frac{10}{7}\right) = \frac{50}{7} = 7.14V$$

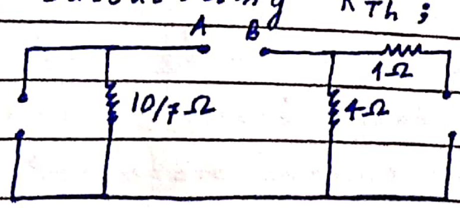
and



$$\therefore \quad V_B = -8V$$

$$\therefore \quad V_{AB} = (7.14) - (-8) = 15.14V = V_{Th}$$

Now, calculating R_{Th};



$$\therefore \quad R_{Th} = \frac{10}{7} + 4 = 5.43\,\Omega$$

©Misbahul Hasan
Scanned with OKEN Scanner

∴ To transfer maximum power;

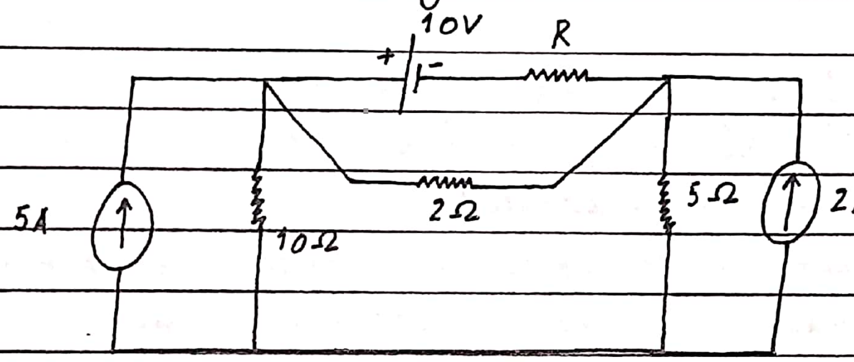
$R_L = R_{th} = 5.43\,\Omega$

According to Maximum Power Transfer Theorem;

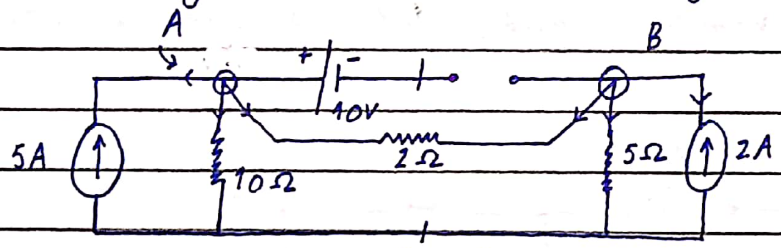
$$P_{Max} = \frac{V_{th}^2}{4R_L} = \frac{(15.14)^2}{4 \times 5.43} = 10.553\ W$$

Ans.

Q. Obtain maximum amount of power in the resistor R from the source using maximum power transfer theorem



Removing 'R' and calculating V_{th}



$V_c = 0$

Applying nodal analysis at 'A'

$$-5 + \frac{V_A}{10} + \frac{V_A - V_B}{2} = 0$$

$$6V_A - 5V_B = 50 \quad \text{(1)}$$

Applying Nodal Analysis at B

$$\therefore \frac{V_B - 10}{2} + \frac{V_B}{5} - 2 = 0$$

$$7V_B - 5V_A = 20 \quad \text{— (2)}$$

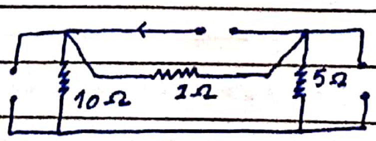
On solving (1) and (2), we get

$V_A = 26.46\ V$

$V_B = 21.76\ V$

$\therefore V_{AB} = 26.46 - 21.76 = 5.3\ V = V_{th}$

Now, calculating R_{Th};



$R_{Th} = (10+5)\,||\,2 = 1.76\,\Omega$

$$\therefore P_{Max} = \frac{V_{th}^2}{4R_L} = \frac{5.3 \times 5.3}{4 \times 1.76 \times 1.76} = 3.990 \approx 4\ W$$

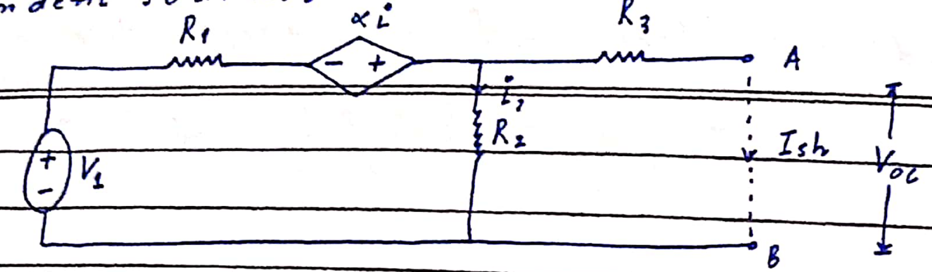
Ans.

©Misbahul Hasan

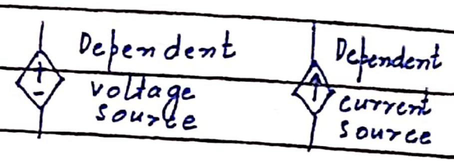
Scanned with OKEN Scanner

Dependent Source

When circuit contains both and dependent and independent sources.



$$R_{th} = \frac{V_{oc}}{I_{sh}}.$$



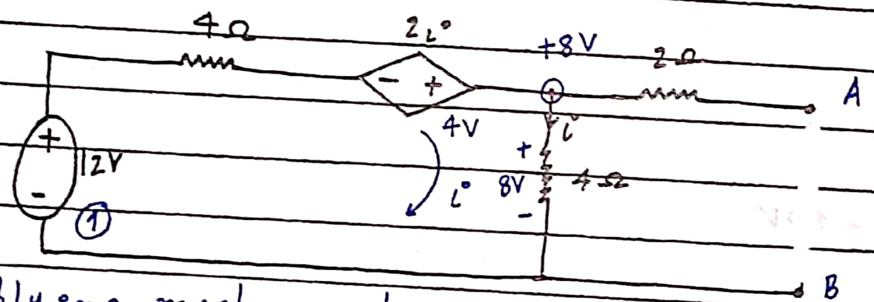
→ Procedure for finding V_{Th} and R_{Th}.

Step 1: Calculate the open circuited voltage V_{oc} as usual with all sources as activated.

Step 2: A short circuit is applied across the open terminal and find the value of short circuited current.

Step 3: Find Thevenin's equivalent resistance.

Q. Find the Thevenin's equivalent network for the given network.



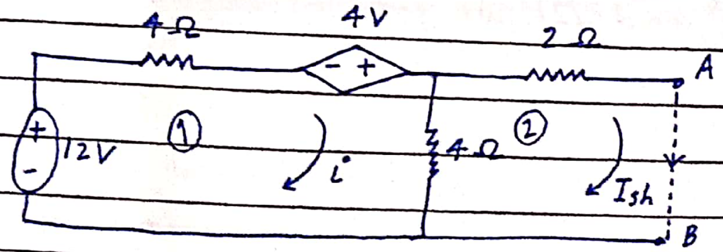
Applying mesh analysis in ①

$\therefore$ $12 - 4i + 2i - 4i = 0$ $\therefore V_{oc} = 8V$

$12 - 6i = 0$

$i = 2$

$\therefore$



Short-circuiting terminal AB and calculating I_{sh} by using mesh analysis

©Misbahul Hasan
Scanned with OKEN Scanner

∴ Applying mesh analysis in 1

$$12 - 4i + 4 - 4(i - i_{sh}) = 0$$

$$16 - 8i + 4i_{sh} = 0 \quad \text{—} (1)$$

Applying mesh analysis in 2

$$4(i_{sh} - i) + 2i_{sh} = 0$$

$$6i_{sh} = 4i$$

$$3i_{sh} = 2i$$

Putting this in 1

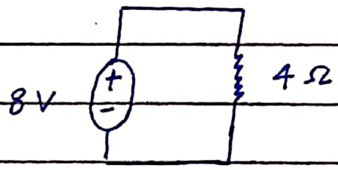
$$16 - 12i_{sh} + 4i_{sh} = 0$$

$$16 - 8i_{sh} = 0$$

$$\boxed{i_{sh} = 2\text{ A}}$$

$$\therefore \quad R_{th} = \frac{V_{oc}}{i_{sh}} = \frac{8}{2} = 4\Omega$$

∴ Thevenin's equivalent network is



Ans.

©Misbahul Hasan

Scanned with OKEN Scanner

These handwritten notes are of ESC-S101 taught to us by Prof. Om Pal, compiled and organized chapter-wise to help our juniors. We hope they make your prep a bit easier.

— **Saksham Nigam** and **Misbahul Hasan** (B.Tech. CSE(2024-28))