Date............
Page............

→ Matrix

Matrix is an arrangement of a mn elements in rectangular array form where 'm' represent rows and 'n' represent columns.

→ Upper Triangular Matrix

A matrix $[a_{ij}]_{m\times n}$ is said to be upper triangular if $a_{ij}=0$ for $i>j$.

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22} & a_{23} \\ 0 & 0 & 0 \end{bmatrix}$$

→ Lower Triangular Matrix

A matrix $[a_{ij}]_{m\times n}$ is said to be lower triangular if $a_{ij}=0$ for $i<j$.

$$A = \begin{bmatrix} a_{11} & 0 & 0 \\ a_{21} & a_{22} & 0 \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

→ Nilpotent Matrix

A matrix $[a_{ij}]_{m\times n}$ is said to be nilpotent if $A^k = 0$. where k is a positive integer.

For example;

$$A = \begin{bmatrix} ab & b^2 \\ -a^2 & -ab \end{bmatrix}$$

$$A^2 = AA = \begin{bmatrix} ab & b^2 \\ -a^2 & -ab \end{bmatrix}\begin{bmatrix} ab & b^2 \\ -a^2 & -ab \end{bmatrix} = \begin{bmatrix} a^2b^2-a^2b^2 & ab^3-ab^3 \\ -a^3b+a^3b & -a^2b^2+a^2b^2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$K = 2.$

Date......
Page......

→ **Idompotent Matrix**

A matrix is said to be idompotent matrix if $A^2 = A$

For example:

$$A = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 2 \\ 1 & -2 & -3 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 2 \\ 1 & -2 & -3 \end{bmatrix} \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 2 \\ 1 & -2 & -3 \end{bmatrix} = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 2 \\ 1 & -2 & -3 \end{bmatrix}$$

→ **Complex Matrix**

A matrix $[a_{ij}]_{m \times n}$ is said to be complex matrix if elements of the matrix (at least one) is in complex form.

For example:

$$A = \begin{bmatrix} 2 & c+id \\ a+ib & 3 \end{bmatrix}$$

→ **Complex Conjugate Matrix**

A matrix $[a_{ij}]_{m \times n}$ is said to be complex conjugate matrix if we occur conjugate of the complex element, i.e

$$a_{ij} = \overline{a_{ij}}$$

→ **Hermitian Matrix**

A complex matrix is said to be Hermitian matrix if $a_{ij} = \overline{a_{ij}}^{T}$.

Date..........
Page..........

For example:

$$A = \begin{bmatrix} 2 & a+ib & c+id \\ a-ib & 3 & e+if \\ c-id & c-if & 0 \end{bmatrix},$$

$$\bar{A} = \begin{bmatrix} 2 & a-ib & c-id \\ a+ib & 3 & c-if \\ c+id & c+if & 0 \end{bmatrix}$$

$$\bar{A}^T = \begin{bmatrix} 2 & a+ib & c+id \\ a+ib & 3 & c+if \\ c+id & c-if & 0 \end{bmatrix}$$

$\Rightarrow \quad \bar{A}^T = A \; ; \quad A \rightarrow$ Hermitian Matrix

→ Skew Hermitian Matrix

A complex matrix is said to be skew hermitian matrix, if $a_{ij} = -\bar{a}_{ij}^T$.

NOTE: The principal diagonal of the skew hermitian matrix either 0 or pure complex.

For example:

$$A = \begin{bmatrix} 0 & a+ib & c+id \\ -(a-ib) & 0 & e+if \\ -(c-id) & -(e-if) & 0 \end{bmatrix}$$

Date.........
Page.........

$\therefore \quad \bar{A} = \begin{bmatrix} 0 & a-ib & c-id \\ -(a+ib) & 0 & e-if \\ -(c+id) & -(e+if) & 0 \end{bmatrix}$

$-\bar{A} = \begin{bmatrix} 0 & -(a-ib) & -(c-id) \\ a+ib & 0 & -(e-if) \\ c+id & e+if & 0 \end{bmatrix}$

$\therefore \quad -\bar{A}^T = \begin{bmatrix} 0 & a+ib & c+id \\ -(a-ib) & 0 & e+if \\ -(c-id) & -(e-if) & 0 \end{bmatrix} = A$

↳ Skew Hermitian Matrix.

→ Rank of a Matrix

Highest order of the non-zero minor of the given matrix is called rank of matrix.

→ Eigenvalues

If $A = [a_{ij}]_{m\times n}$ be a square matrix of order 'n', λ is in independent form with identity matrix, then $A-\lambda I$ is called characteristic matrix.

$$\therefore \quad A - \lambda I = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

If we take determinant of the characteristics matrix we arrive at characteristics polynomial.

$|A - \lambda I| = \begin{vmatrix} a_{11} - \lambda & a_{12} \\ a_{21} & a_{22} - \lambda \end{vmatrix} = \det(A - \lambda I)$

If we equate characteristic polynomial with 0, we obtain characteristics equation.

$\det(A - \lambda I) =$ characteristic polynomial $= 0$

Characteristic equation $= \begin{vmatrix} a_{11} - \lambda & a_{12} \\ a_{21} & a_{22} - \lambda \end{vmatrix} = 0$

If we calculate characteristic equation $|A - \lambda I| = 0$, we get the roots of the characteristics equation, obtained roots are called Eigenvalues or characteristic roots or latent roots.

The set of eigenvalues is called spectrum.

Eigenvetors (Characteristic vector)

Let A be a square matrix,

$$A - \lambda I = 0$$

if X is any vector (column vector), then the eigen-vector is defined as $(A - \lambda I)X = 0$, where O is null matrix.

Date..................
Page..................

→ Solve the differential equations using matrix method

1. $y_1' = -2y_1 + y_2$
$y_2' = y_1 - 2y_2$

$\therefore$ Let

$$Y' = \begin{bmatrix} y_1' \\ y_2' \end{bmatrix},$$

$$Y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \text{ and}$$

$$A = \begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix}$$

$$\therefore \quad Y' = \begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix} Y \quad \text{— ①}$$

Let $y = e^{\lambda t} X$ be a solution of ①, we get

$$\lambda e^{\lambda t} X = A \cdot e^{\lambda t} X$$

$$\therefore \quad \lambda X = AX$$

$$\Rightarrow (A - \lambda I) X = 0$$

Hence,

$$A - \lambda I = \begin{bmatrix} -2-\lambda & 1 \\ 1 & -2-\lambda \end{bmatrix}$$

$\therefore$ The eigenvalues will be given as ;

$$|A - \lambda I| = 0$$

Date.............
Page.............

Hence,

$$(-2-\lambda)^2 - 1 = 0$$
$$4 + \lambda^2 + 4\lambda - 1 = 0$$
$$\lambda^2 + 4\lambda + 3 = 0$$
$$\lambda^2 + 3\lambda + \lambda + 3 = 0$$
$$\lambda(\lambda+3) + 1(\lambda+3) = 0$$
$$(\lambda+1)(\lambda+3) = 0$$
$$\lambda = -1, -3$$

$\therefore$ The eigenvalues are given as $\lambda_1 = -1$ and $\lambda_2 = -3$.

$\Rightarrow$ $$(A - \lambda I)X = 0$$

The eigenvector X_1 corresponding to eigenvalue $\lambda_1 = -1$

$\therefore$ $$\begin{bmatrix} -2-\lambda & 1 \\ 1 & -2-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$\therefore$ $-x_1 + x_2 = 0$ and
$x_1 - x_2 = 0$

$\Rightarrow$ $x_1 = 1$ and $x_2 = 1$

Hence, $X_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

The eigenvector X_2 corresponding to eigenvalue $\lambda_2 = -3$.

Date..........
Page..........

$\therefore \begin{bmatrix} -2-\lambda & 1 \\ 1 & -2-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\therefore \quad x_1 + x_2 = 0$ and
$x_1 + x_2 = 0$

$\Rightarrow \quad x_1 = 1$ and $x_2 = -1$.

Hence, $X_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$\therefore$ The solution to Differential equations :

$$y = e^{\lambda_1 t} x^{(1)} + e^{\lambda_2 t} (x)^{(2)}$$

$$y = e^{-t} \begin{bmatrix} 1 \\ 1 \end{bmatrix} + e^{-3t} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$\therefore \quad y_1 = e^{-t} + e^{-3t}$
$y_2 = e^{-t} - e^{-3t}$. Ans

Date......
Page......

2. $y_1' = 5y_1 + 22y_2$

$y_2' = y_1 + 2y_2$

$\therefore \quad A = \begin{bmatrix} 5 & 22 \\ 1 & 2 \end{bmatrix}$

Hence,

$$Y' = \begin{bmatrix} 5 & 22 \\ 1 & 2 \end{bmatrix} Y$$

To find the eigenvalues ;

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 5-\lambda & 22 \\ 1 & 2-\lambda \end{vmatrix} = 0$$

$$(5-\lambda)(2-\lambda) - 22 = 0$$

$$10 - 5\lambda - 2\lambda + \lambda^2 - 22 = 0$$

$$\lambda^2 - 7\lambda - 12 = 0$$

$$\therefore \quad \lambda = \frac{7 \pm \sqrt{49+48}}{2}$$

$$\lambda = \frac{7 \pm \sqrt{97}}{2}$$

Hence, we obtain eigenvalues ;

$$\lambda_1 = \frac{7}{2} + \frac{\sqrt{97}}{2} \quad \text{and} \quad \lambda_2 = \frac{7}{2} - \frac{\sqrt{97}}{2}$$

$\therefore$ The eigenvector X_1 corresponding to eigenvalue

$$\lambda_1 = \frac{7}{2} + \frac{\sqrt{97}}{2}$$

Date......

Page......

$$\begin{bmatrix} 5-\lambda & 22 \\ 1 & 2-\lambda \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 5-\frac{7}{2}-\frac{\sqrt{97}}{2} & 22 \\ 1 & 2-\frac{7}{2}-\frac{\sqrt{97}}{2} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \frac{3}{2}-\frac{\sqrt{97}}{2} & 22 \\ 1 & -\frac{3}{2}-\frac{\sqrt{97}}{2} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\left(\frac{3}{2}-\frac{\sqrt{97}}{2}\right)x_1 + 22x_2 = 0$$

$$x_1 - \left(\frac{3}{2}+\frac{\sqrt{97}}{2}\right)x_2 = 0$$

$\therefore$ $x_1 = 22$

$x_2 = -\frac{3}{2} + \frac{\sqrt{97}}{2}$

$\therefore$ $$X_1 = \begin{bmatrix} 22 \\ -\frac{3}{2}+\frac{\sqrt{97}}{2} \end{bmatrix}$$

The eigenvector X_2 corresponding to eigenvalue $\lambda_2 = \frac{7}{2} - \frac{\sqrt{97}}{2}$

Date..........
Page..........

$\therefore \begin{bmatrix} 5-\lambda & 22 \\ 1 & 2-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$$\begin{bmatrix} \frac{3}{2}+\frac{\sqrt{97}}{2} & 22 \\ 1 & -\frac{3}{2}+\frac{\sqrt{97}}{2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$\therefore \quad \left(\frac{3}{2}+\frac{\sqrt{97}}{2}\right)x_1 + 22x_2 = 0$

$x_1 + \left(-\frac{3}{2}+\frac{\sqrt{97}}{2}\right)x_2 = 0$

$\therefore \quad x_1 = 22$

$x_2 = -\frac{3}{2} - \frac{\sqrt{97}}{2}$

$\Rightarrow \quad X_2 = \begin{bmatrix} 22 \\ -3/2 - \sqrt{97}/2 \end{bmatrix}$

$\therefore$ The solution to differential equation:

$$y = e^{\lambda_1 t} X^{(1)} + e^{\lambda_2 t} x^{(2)}$$

$$\therefore \quad y = e^{\left(7/2+\frac{\sqrt{97}}{2}\right)t} \begin{bmatrix} 22 \\ -\frac{3}{2}+\frac{\sqrt{97}}{2} \end{bmatrix} + e^{\left(\frac{7-\sqrt{97}}{2}\right)t} \begin{bmatrix} 22 \\ -3/2 - \sqrt{97}/2 \end{bmatrix}$$

$$\Rightarrow \quad y_1 = 22e^{\left(\frac{7+\sqrt{97}}{2}\right)t} + 22e^{\left(\frac{7-\sqrt{97}}{2}\right)t}$$

$$y_2 = e^{\left(\frac{7+\sqrt{97}}{2}\right)}\left(-\frac{3}{2}+\frac{\sqrt{97}}{2}\right) + e^{\left(\frac{7-\sqrt{97}}{2}\right)t}\left(\frac{-3}{2}-\frac{\sqrt{97}}{2}\right)$$

Date......
Page......

3. $$y_1' = 4y_1 + 3y_2$$
$$y_2' = 2y_1 + y_2$$

$$\therefore \quad A = \begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix}$$

$$\therefore \quad Y' = \begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix} Y$$

To get eigenvalues ;

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 4-\lambda & 3 \\ 2 & 1-\lambda \end{vmatrix} = 0$$

$$(4-\lambda)(1-\lambda) - 6 = 0$$
$$4 - 4\lambda - \lambda + \lambda^2 - 6 = 0$$
$$\lambda^2 - 5\lambda - 2 = 0$$
$$\lambda = \frac{5 \pm \sqrt{25+8}}{2}$$
$$\lambda = \frac{5 \pm \sqrt{33}}{2}$$

Hence, we obtain the eigenvalues ;

$\lambda_1 = \dfrac{5 + \sqrt{33}}{2}$ and $\lambda_2 = \dfrac{5 - \sqrt{33}}{2}$

Date.................................
Page.................................

Eigenvector X_1 corresponding to eigenvalue $\lambda_1 = \frac{5}{2} + \frac{\sqrt{33}}{2}$

$\therefore \quad (A - \lambda I)X = 0$

$$\begin{bmatrix} 4-\lambda & 3 \\ 2 & 1-\lambda \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \frac{3}{2} - \frac{\sqrt{33}}{2} & 3 \\ 2 & -\frac{3}{2} - \frac{\sqrt{33}}{2} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\left(\frac{3}{2} - \frac{\sqrt{33}}{2}\right)x_1 + 3x_2 = 0$$

$\therefore \quad x_1 = 3$

$x_2 = -\frac{3}{2} + \frac{\sqrt{33}}{2}$

Hence,

$$X_1 = \begin{bmatrix} 3 \\ -3/2 + \frac{\sqrt{33}}{2} \end{bmatrix}$$

Eigenvector X_2 corresponding to eigenvalue $\lambda_2 = \frac{5}{2} - \frac{\sqrt{33}}{2}$

$\therefore \quad (A - \lambda I)X = 0$

$$\begin{bmatrix} 4-\lambda & 3 \\ 2 & 1-\lambda \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \frac{3}{2} + \frac{\sqrt{33}}{2} & 3 \\ 2 & -\frac{3}{2} + \frac{\sqrt{33}}{2} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\left(\frac{3}{2} + \frac{\sqrt{33}}{2}\right)x_1 + 3x_2 = 0$$

$\therefore$
$$x_1 = 3$$
$$x_2 = -3/2 - \frac{\sqrt{33}}{2}$$

Hence,
$$X_2 = \begin{bmatrix} 3 \\ -3/2 - \sqrt{33}/2 \end{bmatrix}$$

$\therefore$ The solution of differential equation;

$$y = e^{\lambda_1 t} x^{(1)} + e^{\lambda_2 t} x^{(2)}$$

$$y = e^{\left(5/2 + \frac{\sqrt{33}}{2}\right)t} \begin{bmatrix} 3 \\ -3/2 + \frac{\sqrt{33}}{2} \end{bmatrix} + e^{\left(5/2 - \frac{\sqrt{33}}{2}\right)t} \begin{bmatrix} 3 \\ -3/2 - \frac{\sqrt{33}}{2} \end{bmatrix}$$

$\therefore$
$$y_1 = 3\, e^{\left(\frac{5+\sqrt{33}}{2}\right)t} + 3\, e^{\left(\frac{5-\sqrt{33}}{2}\right)t}$$

$$y_2 = e^{\left(\frac{5+\sqrt{33}}{2}\right)t}\left(\frac{-3+\sqrt{33}}{2}\right) + 3\, e^{\left(\frac{5-\sqrt{33}}{2}\right)t}\left(\frac{-3-\sqrt{33}}{2}\right).$$ Ans.

Date.......................................
Page.......................................

4.

$$y_1' = 3y_1 + 2y_2$$
$$y_2' = y_1 - y_2$$

$$A = \begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix}$$

$$\therefore \quad Y' = \begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix} Y$$

To obtain eigenvalues;

$$|A - \lambda I| = 0$$

$$\det \begin{bmatrix} 3-\lambda & 2 \\ 1 & -1-\lambda \end{bmatrix} = 0$$

$$(3-\lambda)(-1-\lambda) - 2 = 0$$
$$-3 - 3\lambda + \lambda + \lambda^2 - 2 = 0$$
$$\lambda^2 - 2\lambda - 5 = 0$$

$$\lambda = \frac{2 \pm \sqrt{4+20}}{2}$$

$$\lambda = \frac{2 \pm \sqrt{24}}{2}$$

$$\lambda = \frac{2 \pm 2\sqrt{6}}{2} = 1 \pm \sqrt{6}$$

Hence, we obtain eigenvalues;

$\lambda_1 = 1 + \sqrt{6}$ and $\lambda_2 = 1 - \sqrt{6}$

$\therefore$ Eigenvector X_1 corresponding to eigenvalue $\lambda_1 = 1+\sqrt{6}$

$\therefore$ $(A-\lambda I)X = 0$

$$\begin{bmatrix} 3-\lambda & 2 \\ 1 & -1-\lambda \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 2-\sqrt{6} & 2 \\ 1 & -2-\sqrt{6} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$(2-\sqrt{6})x_1 + 2x_2 = 0$$
$$x_1 - x_2(2+\sqrt{6}) = 0$$

$\therefore$ $x_1 = 2$

$x_2 = -2+\sqrt{6}$

$$X_1 = \begin{bmatrix} 2 \\ -2+\sqrt{6} \end{bmatrix}$$

$\therefore$ Eigenvector X_2 corresponding to eigenvalue $\lambda_2 = 1-\sqrt{6}$

$\therefore$ $(A-\lambda I)X = 0$

$$\begin{bmatrix} 3-\lambda & 2 \\ 1 & -1-\lambda \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 2+\sqrt{6} & 2 \\ 1 & -2+\sqrt{6} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$(2+\sqrt{6})x_1 + 2x_2 = 0$$
$$x_1 + (-2+\sqrt{6})x_2 = 0$$

$x_1 = 2$ and $x_2 = (-2-\sqrt{6})$

Hence $X_2 = \begin{bmatrix} 2 \\ -2-\sqrt{6} \end{bmatrix}$

$\therefore$ The solution to differential equation :

$$y = e^{\lambda_1 t} x^{(1)} + e^{\lambda_2 t} x^{(2)}$$

$$y = e^{(1+\sqrt{6})t} \begin{bmatrix} 2 \\ -2+\sqrt{6} \end{bmatrix} + e^{(1-\sqrt{6})t} \begin{bmatrix} 2 \\ -2-\sqrt{6} \end{bmatrix}$$

$\therefore$

$$y_1 = 2e^{(1+\sqrt{6})t} + 2e^{(1-\sqrt{6})t}$$

$$y_2 = e^{(1+\sqrt{6})t}(-2+\sqrt{6}) + e^{(1-\sqrt{6})t}(-2-\sqrt{6})$$ Ans.

Date.................
Page.................

Scanned with OKEN Scanner

Date.................
Page.................

5.

$$y_1' = y_1 + 3y_2$$
$$y_2' = 2y_1 + 2y_2$$

$\therefore$ $$A = \begin{bmatrix} 1 & 3 \\ 2 & 2 \end{bmatrix}$$

$\therefore$ $$Y' = \begin{bmatrix} 1 & 3 \\ 2 & 2 \end{bmatrix} Y$$

To obtain eigenvalues:

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 1-\lambda & 3 \\ 2 & 2-\lambda \end{vmatrix} = 0$$

$$(1-\lambda)(2-\lambda) - 6 = 0$$
$$2 - \lambda - 2\lambda + \lambda^2 - 6 = 0$$
$$\lambda^2 - 3\lambda - 4 = 0$$
$$\lambda = \frac{3 \pm \sqrt{9+16}}{2}$$
$$\lambda = 4, -1.$$

We obtain eigenvalues $\lambda_1 = 4$ and $\lambda_2 = -1$

Hence, eigenvector X_1 corresponding to eigenvalue $\lambda_1 = 4$

$$(A - \lambda I)X = 0$$

$$\begin{bmatrix} -3 & 3 \\ 2 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$-3x_1 + 3x_2 = 0$$
$$2x_1 - 2x_2 = 0$$

$\therefore$ $x_1 = 1$ and $x_2 = 1$

$$X_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Hence, eigenvector X_2 corresponding to eigenvalue $\lambda_2 = -1$

Date........
Page........

$$(A - \lambda I)X = 0$$

$$\begin{bmatrix} 2 & 3 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$2x_1 + 3x_2 = 0$$

$$x_1 = 3 \quad \text{and} \quad x_2 = -2$$

$$\therefore \quad X_2 = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$$

Hence, the solution to differential equation :

$$y = e^{\lambda_1 t} x^{(1)} + e^{\lambda_2 t} x^{(2)}$$

$$\therefore \quad y = e^{4t}\begin{bmatrix} 1 \\ 1 \end{bmatrix} + e^{-t}\begin{bmatrix} 3 \\ -2 \end{bmatrix}$$

$$y_1 = e^{4t} + 3e^{-t}$$

$$y_2 = e^{4t} - 2e^{-t}.$$ Ans.

6.

$$y_1' = 2y_1 + y_2$$

$$y_2' = y_1 + 3y_2$$

$$\therefore \quad A = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$$

$$\therefore \quad Y' = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix} Y$$

To obtain eigenvalues ;

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 2-\lambda & 1 \\ 1 & 3-\lambda \end{vmatrix} = 0$$

$$(2-\lambda)(3-\lambda) - 1 = 0$$

$$6 - 2\lambda - 3\lambda + \lambda^2 - 1 = 0$$

$$\lambda^2 - 5\lambda + 5 = 0$$

$$\lambda = \frac{5 \pm \sqrt{25-20}}{2} = \frac{5 \pm \sqrt{5}}{2}$$

$\therefore$ We obtain eigenvalues $\lambda_1 = \frac{5+\sqrt{5}}{2}$ and $\lambda_2 = \frac{5-\sqrt{5}}{2}$.

$\Rightarrow$ Eigenvector X_1 corresponding to eigenvalue $\lambda_1 = \frac{5+\sqrt{5}}{2}$.

$$(A - \lambda I)X = 0$$

$$\begin{bmatrix} -\frac{1}{2} - \frac{\sqrt{5}}{2} & 1 \\ 1 & \frac{1}{2} - \frac{\sqrt{5}}{2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$x_1\left(-\frac{1}{2} - \frac{\sqrt{5}}{2}\right) - x_2 = 0$$

$$\therefore \quad x_1 + x_2\left(\frac{1}{2} - \frac{\sqrt{5}}{2}\right) = 0$$

Hence $x_1 = 1$, $x_2 = -\frac{1}{2} - \frac{\sqrt{5}}{2}$

$$\therefore \quad X_1 = \begin{bmatrix} 1 \\ -\frac{1}{2} - \frac{\sqrt{5}}{2} \end{bmatrix}$$

$\Rightarrow$ Eigenvector X_2 corresponding to eigenvalue $\lambda_2 = \frac{5-\sqrt{5}}{2}$.

$$(A - \lambda I)X = 0$$

$$\begin{bmatrix} -\frac{1}{2} + \frac{\sqrt{5}}{2} & 1 \\ 1 & \frac{1}{2} + \frac{\sqrt{5}}{2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\left(-\frac{1}{2} + \frac{\sqrt{5}}{2}\right)x_1 + x_2 = 0$$

$\therefore$ $x_1 = 1$ and $x_2 = \frac{1}{2} - \frac{\sqrt{5}}{2}$

Date.......................
Page.......................

$$X_2 = \begin{bmatrix} 1 \\ 1/2 - \sqrt{5}/2 \end{bmatrix}$$

$\therefore$ The solution of differential equation;

$$y = e^{\lambda_1 t} x^{(1)} + e^{\lambda_2 t} x^{(2)}$$

$$y = e^{\left(\frac{5+\sqrt{5}}{2}\right)t} \begin{bmatrix} 1 \\ -1/2 - \sqrt{5}/2 \end{bmatrix} + e^{\left(\frac{5-\sqrt{5}}{2}\right)t} \begin{bmatrix} 1 \\ 1/2 - \sqrt{5}/2 \end{bmatrix}$$

$$y_1 = e^{\left(\frac{5+\sqrt{5}}{2}\right)t} + e^{\left(\frac{5-\sqrt{5}}{2}\right)t}$$

$$y_2 = e^{\left(\frac{5+\sqrt{5}}{2}\right)t} \left[-1/2 - \sqrt{5}/2\right] + e^{\left(\frac{5-\sqrt{5}}{2}\right)t} \left[1/2 - \sqrt{5}/2\right]$$ Ans.

7. $$y_1' = -2y_1 + 3y_2$$
$$y_2' = 4y_1 - y_2$$

$\therefore$ $$A = \begin{bmatrix} -2 & 3 \\ 4 & -1 \end{bmatrix}$$

$$Y' = \begin{bmatrix} -2 & 3 \\ 4 & -1 \end{bmatrix} Y$$

$\therefore$ For eigenvalues;

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} -2-\lambda & 3 \\ 4 & -\lambda - 1 \end{vmatrix} = 0$$

$$(-2-\lambda)(-\lambda-1) - 12 = 0$$

$$2\lambda + 2 + \lambda^2 + \lambda - 12 = 0$$

$$\lambda^2 + 3\lambda - 10 = 0$$

$$\lambda^2 + 5\lambda - 2\lambda - 10 = 0$$

$$\lambda(\lambda+5) - 2(\lambda+5) = 0$$
$$(\lambda+5)(\lambda-2) = 0$$
$$\lambda = -5, 2$$

$\therefore$ We obtained Eigenvalues ; $\lambda_1 = -5$ and $\lambda_2 = 2$

Hence, eigenvector X_1 corresponding to eigenvalue $\lambda_1 = -5$.

$$(A - \lambda I)X = 0$$
$$\begin{bmatrix} 3 & 3 \\ 4 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$
$$3x_1 + 3x_2 = 0$$
$\therefore$ $x_1 = 1$ and $x_2 = -1$

Hence, $X_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

and; eigenvector X_2 corresponding to eigenvalue $\lambda_2 = 2$

$$(A - \lambda I)X = 0$$
$$\begin{bmatrix} -4 & 3 \\ 4 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$
$\therefore$ $-4x_1 + 3x_2 = 0$

$x_1 = 3$ and $x_2 = 4$

Hence, $X_2 = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$

$\therefore$ The solution of differential equation ;

$$y = e^{\lambda_1 t} x^{(1)} + e^{\lambda_2 t} x^{(2)}$$
$$y = e^{-5t} \begin{bmatrix} 1 \\ -1 \end{bmatrix} + e^{2t} \begin{bmatrix} 3 \\ 4 \end{bmatrix}$$
$$y_1 = e^{-5t} + 3e^{2t}$$
$$y_2 = -e^{-5t} + 4e^{2t}.$$ Ans.