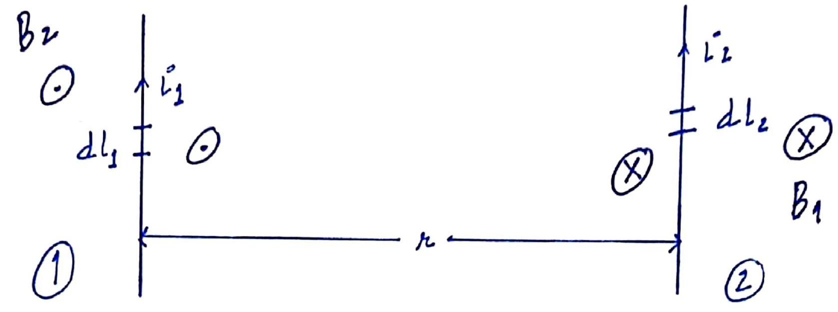
Force between 2 finite element of current



Let us consider two finite elements dl_1 and dl_2 with currents i_1 and i_2 respectively.

$\therefore$ Magnetic field due to dl_1 on wire 2 will be given as:

$$dB_1 = \frac{\mu_0}{4\pi} \frac{i_1 \, dl_1 \sin 90^\circ}{r^2}$$ (As, r and dl are $\perp^r$ to each other)

$$dB_1 = \frac{\mu_0}{4\pi} \frac{i_1 \, dl_1}{r^2}$$

Hence, Force experienced by wire 2 due to this magnetic field.

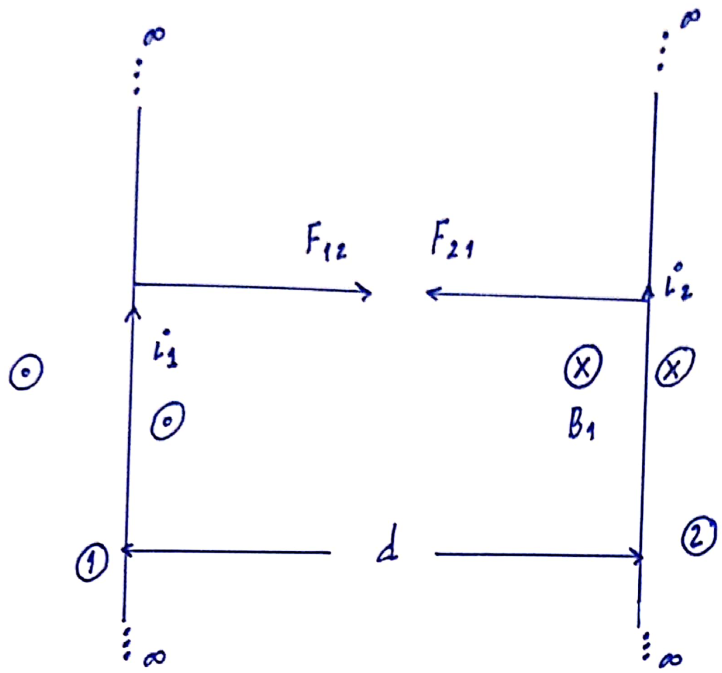
$$d\vec{F}_{21} = i_2 \left(d\vec{l}_2 \times \vec{B}_1 \right)$$

$$d\vec{F}_{21} = i_2 \left[\frac{\mu_0}{4\pi} \frac{i \, d\vec{l}_1}{r^2} \times d\vec{l}_2 \right]$$

$$\boxed{d\vec{F}_{21} = \frac{\mu_0}{4\pi} \frac{i_1 i_2}{r^2} \left(d\vec{l}_1 \times d\vec{l}_2 \right).}$$

As, the currents are flowing in the same direction, the nature of force will be attractive.

Interaction between two parallel wires



Let us consider two wires ① and ② carrying currents i_1 and i_2 respectively, seperated by distance 'd'.

Magnetic Field due to wire ① (B_1) will be inside the plane/paper and will be given as;

$$\vec{B_1} = \frac{\mu_0 2 i_1}{4\pi d}$$

∴ Force experienced by wire ② due to this magnetic field will be given as;

$$\vec{F_{21}} = i_2 B_1 l$$

Force on ② due to ①

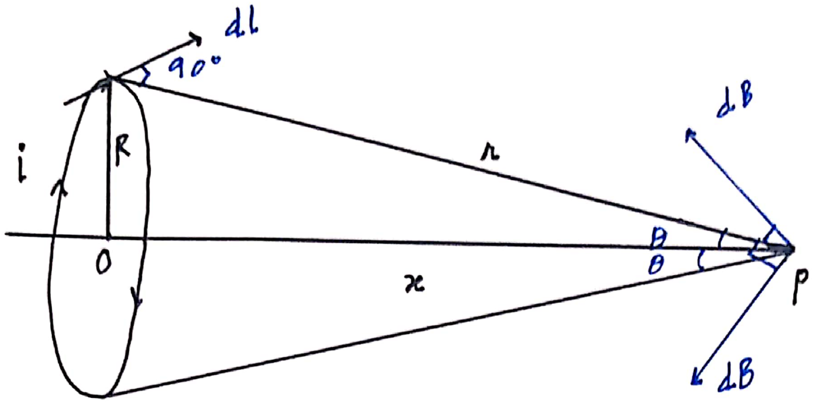
$$\therefore \vec{F_{21}} = i_2 \frac{\mu_0 2 i_1}{4\pi d} \times l$$

$$\Rightarrow \boxed{\frac{\vec{F_{21}}}{l} = \frac{\mu_0}{4\pi} \frac{2 i_1 i_2}{d}}$$

As, the currents are flowing in the same direction, thus force would be attractive in nature.

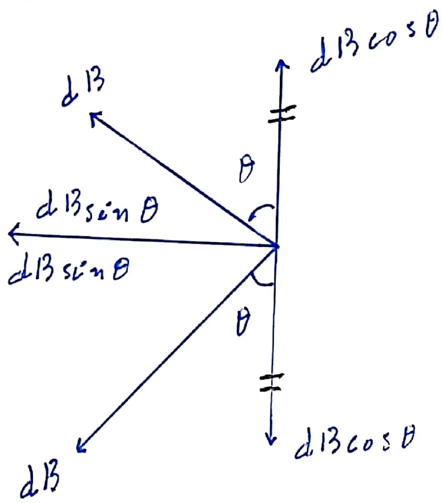
Similar can be evaluated for $\vec{F_{12}}$.

Magnetic field along the axis of circular coil



Let us consider a circular coil of radius 'R' carrying a steady current 'i'.

Now, considering a small amount/element 'dl', the magnetic field formed at point P would be given as;



and, according to Biot Savart's Law;

$$dB = \frac{\mu_0}{4\pi} \frac{i\,dl \sin 90^\circ}{r^2}$$

But, magnetic field due to complete coil will be given as;

$$\vec{B} = \int dB \sin\theta$$

$$\sin\theta = \frac{R}{r}$$

$$\vec{B} = \int \frac{\mu_0}{4\pi} \frac{i\,dl}{r^2} \times \frac{R}{r}$$

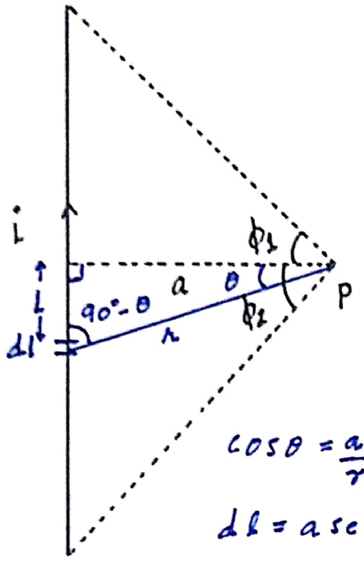
$$r = (R^2 + x^2)^{1/2}$$

$$\vec{B} = \frac{\mu_0 i R}{4\pi r^3} \cdot 2\pi R$$

$$\vec{B} = \frac{\mu_0 i R^2}{2(R^2+x^2)^{3/2}}.$$

Magnetic field due to steady current in a long straight wire.

Let us consider a long straight wire which carries a steady current 'i'.



Magnetic field due to small element 'dl' at point P.

According to Biot-Savart's Law:

$$\vec{dB} = \frac{\mu_0}{4\pi} \frac{i\,dl \sin(90-\theta)}{r^2}$$

$$\vec{dB} = \frac{\mu_0}{4\pi} \frac{i\,dl\cos\theta}{r^2}$$

$\cos\theta = \frac{a}{r} \Rightarrow \frac{a}{\cos\theta} = r$

$dl = a\sec^2\theta$

Now, integrating 'dB':

$$\therefore \int \vec{dB} = \frac{\mu_0}{4\pi}\int_{-\phi_2}^{+\phi_1} \frac{i\,a\sec^2\theta\,d\theta\cos\theta}{\frac{a^2}{\cos^2\theta}}$$

$$\int \vec{dB} = \frac{\mu_0}{4\pi}\frac{i}{a}\int_{-\phi_2}^{\phi_1}\cos\theta\,d\theta$$

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{i}{a}\left[\sin\theta\right]_{-\phi_2}^{\phi_1}$$

$$\boxed{\vec{B} = \frac{\mu_0}{4\pi}\frac{i}{a}\left[\sin\phi_1 + \sin\phi_2\right]}.$$

Curl of Magnetic Field

According to Ampere's Law, the line integral of magnetic induction $\vec{B}$ along a closed path in a magnetic field due to an electric current is equal to μ_0 times total current enclosed by the closed path.

$$\oint \vec{B}\cdot\vec{dl} = \mu_0 I = \mu_0 \int_S \vec{J}\cdot d\vec{S} \quad \text{— (1)}$$

According to Stoke's Theorem

$$\oint \vec{B}\cdot\vec{dl} = \int_S (\mathrm{curl}\,\vec{B})\cdot dS \quad \text{— (2)}$$

Hence, from eq (1) and eq (2)

$$\int_S (\mathrm{curl}\,\vec{B})\cdot d\vec{S} = \mu_0 I = \mu_0 \int_S \vec{J}\cdot d\vec{S} \quad \text{— (3)}$$

or

$$\boxed{\begin{array}{c} \mathrm{curl}\,\vec{B} = \mu_0 \vec{J} \\ \vec{\nabla}\times\vec{B} = \mu_0 \vec{J} \end{array}} \quad \text{— (4)}$$

This equation valid for steady current for varying electric field.