Displacement Current

$$I_d = \varepsilon_0 \frac{d\phi_E}{dt}$$

→ Faraday's Law of Electromagnetic Induction

First Law: Emf is induced due to change in magnetic flux.

$$\Rightarrow \quad e = -\frac{d\phi_B}{dt}$$

Second Law (Lenz's Law): Induced emf opposes the change in magnetic flux.

→ Vector form of Faraday's Law (Differential and Integral).

1. Integral form:

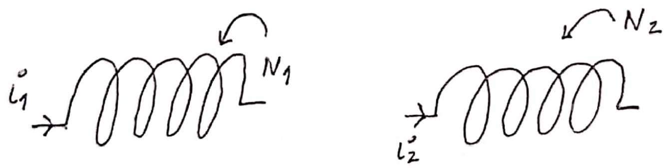
$$\oint \vec{E}\cdot d\vec{l} = -\frac{d}{dt}\int_S \vec{B}\cdot d\vec{s}$$

2. Differential form:

$$\text{curl } \vec{E} = -\frac{d\vec{B}}{dt}$$

$$\vec{\nabla}\times\vec{E} = -\frac{d\vec{B}}{dt}$$

→ Mutual Inductance



$$\therefore \quad N_2\phi_2 \propto i_1$$

$$N_2\phi_2 = M i_1$$

$$M = \frac{N_2\phi_2}{i_1}$$

$$\therefore \quad e_1 = -M\frac{di_2}{dt}$$

Mutual Inductance of a solenoid

$$M = \mu_0 n_1 n_2 \pi r^2 l$$

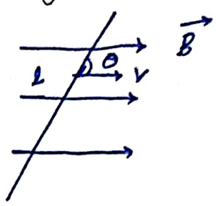
l → length of solenoid

r → radius of solenoid

→ EMF due to movement of a conducting rod

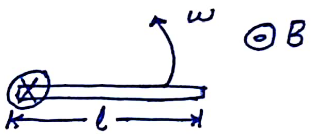
$$e = B(\vec{v} \times \vec{l})$$

$$e = Bvl\sin\theta$$



→ EMF due to rotation of a conducting rod.

$$\therefore \quad e = \frac{1}{2}B\omega l^2$$



$v = l\omega$

$\therefore \omega = \frac{l}{v}$

→ Maxwell's Equation

The divergence and curl relation of electromagnetic fields are called Maxwell Equations.

→ Integral Form

1. $\oint \vec{E} \cdot d\vec{s} = \frac{q}{\epsilon_0}$ [Gauss's Law in Electrostats]

2. $\oint \vec{B} \cdot d\vec{s} = 0$ [Gauss's Law in Magnetostats]

3. $\oint \vec{E} \cdot d\vec{l} = -\frac{d\phi_B}{dt}$ [Faradday's Law of EMI]

4. $\oint \vec{B} \cdot d\vec{l} = \mu_0 (\underbrace{i}_{\text{traditional current}} + \underbrace{\epsilon_0 \frac{d\phi_E}{dt}}_{\text{displacement current}})$ [Modified Ampere circuital law]

→ Differential Form

1. $div\ E = +\frac{\rho_0}{\epsilon_0}$ or $Div\ D = \rho$

2. $div\ B = 0$

3. $curl\ \vec{E} = -\frac{\partial B}{\partial t}$

4. $curl\ B = \mu_0 (\vec{J} + \frac{\partial \vec{D}}{\partial t}) = \mu_0 (\vec{J} + \epsilon_0 \frac{d\vec{E}}{dt})$

Maxwell's equation for free space

1. $\vec{\nabla}.\vec{E} = 0$
2. $\vec{\nabla}.\vec{B} = 0$
3. $\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}$
4. $\vec{\nabla}\times\vec{B} = \mu_0 \epsilon \frac{\partial \vec{E}}{\partial t}$

→ Electromagnetic Wave Equation in medium

$$\nabla^2\vec{E} - \sigma\mu\frac{\partial \vec{E}}{\partial t} - \varepsilon\mu\frac{\partial^2 \vec{E}}{\partial t^2} = 0$$

$$\nabla^2\vec{H} - \sigma\mu\frac{\partial \vec{H}}{\partial t} - \varepsilon\mu\frac{\partial^2 \vec{H}}{\partial t^2} = 0$$

→ EMW equation in vacuum

$$\left.\begin{array}{l} \sigma = 0 \\ \varepsilon = \varepsilon_0 \\ \mu = \mu_0 \end{array}\right\} \text{For vacuum}$$

$$\nabla^2\vec{E} - \mu_0\varepsilon_0\frac{\partial^2 \vec{E}}{\partial t^2} = 0$$

$$\nabla^2\vec{H} - \mu_0\varepsilon_0\frac{\partial^2 \vec{H}}{\partial t^2} = 0$$

Above equation can be written as

$$\nabla^2 U - \frac{1}{v^2}\frac{\partial^2 U}{\partial t^2} = 0$$

$$\Rightarrow \quad v = \frac{1}{\sqrt{\varepsilon_0\mu_0}} = 3\times10^8 = c$$

and EMW equation will be given as; (for v

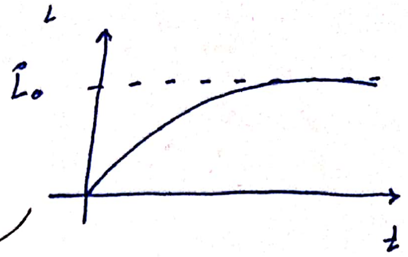
$$\nabla^2\vec{E} - \frac{1}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2} = 0$$

$$\nabla^2\vec{H} - \frac{1}{c^2}\frac{\partial^2 \vec{H}}{\partial t^2} = 0$$

$$\nabla^2 U - \frac{1}{c^2}\frac{\partial^2 U}{\partial t^2} = 0$$

Poyting Vector

$$\vec{S} = \frac{1}{\mu_0}(\vec{E} \times \vec{B})$$



→ Growth of current in LR circuit

$$i = i_0 (1 - e^{-R/L\, t})$$

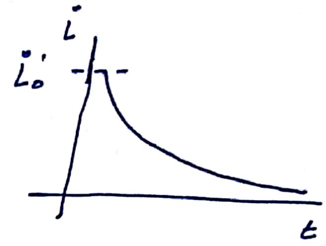
$$\frac{L}{R} = \lambda$$

$$i = i_0 (1 - e^{-t/\lambda})$$

inductive time constant

λ → time that current takes to grows from 0 to 63%

→ Decay of current



$$i = i_0 e^{-R/L\, t}$$

$$\frac{L}{R} = \lambda$$

$$i = i_0 e^{-t/\lambda}$$

λ → time that current takes to decay from 0 to 37%

→ Relation between B_0 and E_0

$$E = Bc$$

$$\therefore \; E = B\underbrace{V}_{\text{velocity of wave}}$$

$E = E_0 \sin(\omega t - \beta x)$

↳ Wave propogating in $+x$ direction.

→ $$V = \frac{1}{\sqrt{\mu_r \varepsilon_r}}$$

→ Instantaneous Energy density $= \varepsilon_0 E^2 = \dfrac{B^2}{\mu_0}$

→ Average energy density $= \dfrac{1}{2}\varepsilon_0 E^2 = \dfrac{B^2}{2\mu_0}$

→ Wavelength : $\lambda = \dfrac{V \text{ (velocity)}}{f \text{ (frequency)}} = \dfrac{2\pi}{\beta \text{ (phase constant)}}$

→ Phase constant (β) : $\dfrac{2\pi}{\lambda}$.

→ Intrinsic Independence (η) = $\eta = \eta_0 \sqrt{\dfrac{\mu_r}{\varepsilon_r}}$ $\eta_0 = 333\Omega$ for vacuum

→ Electric Field Intensity $E_0 = \eta H_0$

These handwritten notes are of PHY-S102 taught to us by Prof. Prabal Pratap Singh, compiled and organized chapter-wise to help our juniors. We hope they make your prep a bit easier.

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