Gauss's Divergence Theorem

$$\iint_S (\vec{F}\cdot\hat{n})\, dS = \iiint_V (\underbrace{\vec{\nabla}\cdot\vec{F}}_{\text{Divergence of } F})\, dV$$

Gauss's Divergence Theorem relates 'surface integral' and 'volume integral'.

Stoke's Theorem

$$\underbrace{\oint \vec{F}\cdot d\vec{l}}_{\text{Line Integral}} = \underbrace{\int_S}_{\text{Surface integral}} (\overbrace{\vec{\nabla}\times\vec{F}}^{\text{curl of } F})\, dS$$

Coulomb's Law

$$F = \frac{K q_1 q_2}{r^2} \qquad K = \frac{1}{4\pi\epsilon_0} = 9\times10^9 \ \frac{Nm^2}{C}$$

$$\vec{F} = \frac{K q_1 q_2 \vec{r}}{|r|^3} \ \begin{bmatrix}\text{Vector}\\ \text{Form}\end{bmatrix} \qquad \epsilon_0 = 8.85\times10^{-12} \ \frac{C}{Nm^2}$$

Electric Field Intensity

$$\vec{E} = \frac{\vec{F}}{q_0}$$

Unit → N/C or V/m

Dimensional Formula : $[MLT^{-3}A^{-1}]$

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$$

→ Electric field due to line charge distribution

$$E = \frac{1}{4\pi\epsilon_0}\frac{\int \lambda\, dl\, (\vec{r}-\vec{r_0})}{|r-r_0|^3}$$

→ Electric field due to surface charge distribution

$$E = \frac{1}{4\pi\epsilon_0}\frac{\int \sigma\, dA\, (\vec{r}-\vec{r_0})}{|r-r_0|^3}$$

(iii) Due to volume charge distribution

$$E = \frac{1}{4\pi\epsilon_0} \frac{\int \rho \, dV}{|r - r_0|^3} (\vec{r} - \vec{r_0})$$

→ Electrostatic Potential

$$V = \frac{W}{q_0} = \frac{F \cdot r}{q_0} = \frac{1}{4\pi\epsilon_0} \frac{q}{r} \quad V$$

Unit : Volt, Joule / Coulomb, Newton-metre / Coulomb

Dimensional Formula : $[ML^2T^{-3}A^{-1}]$

→ Potential and Electric Field

$$E = -\frac{dV}{dx}$$

and $$\vec{E} = -\nabla V = -\text{grad}V$$

→ Curl of $\vec{E}$

For closed path ;

$$\vec{\nabla} \times \vec{E} = \text{curl}\vec{E} = 0.$$

→ Electric Flux

$$\phi_E = \oint \vec{E} \cdot d\vec{A}$$

Unit : $\frac{Nm^2}{C}$, Vm

$$\phi_E = EA\cos\theta$$

Dimensional formula : $[ML^3T^{-3}A^{-1}]$

→ Electric Flux Density

$$\therefore \frac{\phi_E}{A} = \frac{EA}{A} = E$$

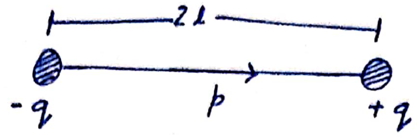
→ Gauss's Law (Integral Form)

$$\phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{q}{\epsilon_0}$$

→ Gauss's Law (Differential Form)

$$\text{div}\vec{E} = \vec{\nabla} \cdot \vec{E} = -\frac{\rho}{\epsilon_0}$$

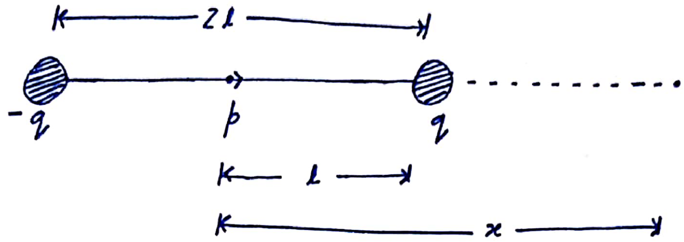
→ Electric Dipole



$p = q \times 2l$

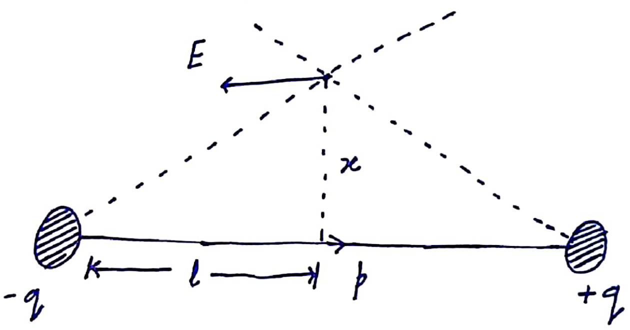
Unit : Coulomb-meter

→ Electric field due to dipole at axial position:



$$\therefore \quad \vec{E} = \frac{2K\vec{p}}{x^3}$$

→ Electric field due to dipole at an equitorial position:



$$\therefore \quad \vec{E} = \frac{K\vec{p}}{x^3}$$

→ Torque on dipole in an electric field.

$$\vec{\tau} = \underbrace{\vec{P}}_{\text{dipole moment}} \times \vec{E} = PE\sin\theta$$

→ Potential Energy of dipole

$$U = -\vec{P} \cdot \vec{E} = -PE\cos\theta$$

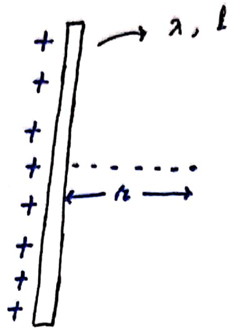
→ Work done by dipole

$$\therefore \quad W = \Delta U = U_f - U_i$$

$$W = PE(\cos\theta_i - \cos\theta_f).$$

Application of Gauss Law

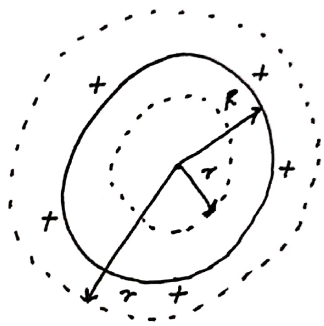
1. Electric field due to an infinite line of charge



$$E = \frac{\lambda}{2\pi\epsilon_0 r} = \frac{2k\lambda}{r}$$

2. Electric Field due to a charged spherically shell

$$Q = 4\pi R^2 \sigma$$



CASE 1:

∴ If $r > R$

$$E = \frac{\sigma}{\epsilon_0} \cdot \frac{R^2}{r^2}$$

CASE 2:

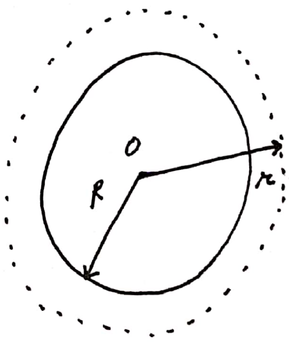
If $r = R$

$$E = \frac{\sigma}{\epsilon_0}$$

CASE 3: $r < R$

$$E = 0 \qquad (\sigma = 0)$$

3. Electric field of a uniformly charged sphere



CASE 1: If $r > R$;

$$E = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2}$$

CASE 2: If $r = R$;

$$E = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{R^2}$$

CASE 3: If $r < R$;

(a) If sphere is uniformly charged sphere conducting

$$E = 0$$

(b) If sphere is non-conducting

$$E = \frac{1}{4\pi\epsilon_0} \frac{qr}{R^3} .$$

→ Poisson's Equation:

$$\text{div}(\text{grad } V) = \frac{\rho}{\varepsilon_0}$$

$$\therefore \quad \nabla(\nabla V) = \frac{\rho}{\varepsilon_0}$$

$$\boxed{\nabla^2 V = \frac{\rho}{\varepsilon_0}}$$

↳ Laplacian operator.

→ Laplace Equation:

$$\boxed{\nabla^2 V = 0}$$

Current (I)

$$I = \frac{dq}{dt} \quad \text{Ampere}$$

→ Linear Current Density (I)

$$dq = \lambda d\vec{l}$$

$$\therefore \quad I = \frac{\lambda d\vec{l}}{dt} = \lambda\vec{v}$$

$$\therefore \quad F_m = \int(\vec{v}\times\vec{B})\,dq = \int(\vec{v}\times\vec{B})\lambda d\vec{l}$$

$$F_m = \int(\vec{I}\times\vec{B})\,dl$$

→ Surface Current Density (K)

$$dq = \sigma\, dl$$

$$\therefore \quad I = \frac{\sigma\, dl}{dt} = \sigma\vec{v}$$

$$F_m = \iint(\vec{v}\times\vec{B})\,dq = \iint(\vec{v}\times\vec{B})\sigma\, ds$$

$$F_m = \iint(\vec{K}\times\vec{B})\,ds$$

→ Volume Current Density ($\vec{J}$)

$$dq = \rho\, dl$$

$$\therefore \quad I = \frac{\rho\, dl}{dt} = \rho\vec{v}$$

$$F_m = \iiint(\vec{v}\times B)\,dq = \iiint(\vec{v}\times B)\rho\, dV$$

$$F_m = \iiint(\vec{J}\times\vec{B})\,dV$$

→ Magnetic Field ($\vec{B}$)

$1 \text{ Gauss} = 10^{-4}\,T$

Unit : Tesla, $\frac{\text{Weber}}{m^2}$, Gauss

Dimensional Formula : $[MA^{-1}T^{-2}]$

→ Lorentz Force

$$F_L = F_E + F_M$$
$$= qE + q(\vec{v} \times \vec{B})$$
$$F_L = q(E + (\vec{v} \times \vec{B})).$$

→ Magnetic Flux (ϕ_B)

$$\phi_B = B S \cos\theta = \vec{B} \cdot \vec{S}$$

Unit : SI - Weber or Tesla-m^2

CGS - Maxwell or Gauss-cm^2

1 Weber = 10^8 Maxwell

Dimensional Formula : $[ML^2A^{-1}T^{-2}]$

→ Gauss's Law in Magnetostats :

$$\therefore \oint \vec{B} d\vec{s} = 0$$

Using Divergence theorem

$$\int B \cdot dS = \int (\vec{\nabla} \cdot \vec{B}) dV$$

$$\therefore \boxed{div\, B = \vec{\nabla} \cdot \vec{B} = 0}$$

→ Torque on a current coil in magnetic field

$$\tau = (NIA) B \sin\theta$$
$$\tau = MB \sin\theta$$
$$\vec{\tau} = \vec{M} \times \vec{B}$$

$M = NIA$

↳ Magnetic dipole moment

Unit : Ampere-m^2 or $\frac{\text{Joule}}{\text{Tesla}}$

→ Magnetic Dipole Moment

$$m = q_m \times 2l$$

↳ Pole strength

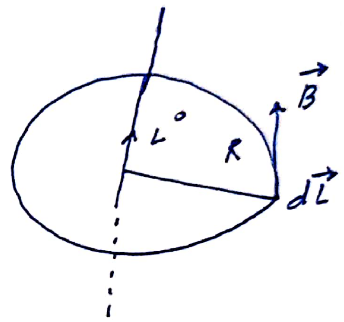
→ Potential energy of a magnetic dipole

$$U = -\vec{M} \cdot \vec{B}$$

$$U = -MB\cos\theta$$

→ Biot-Savart Law;

$$dB = \frac{\mu_0}{4\pi} \frac{i(\vec{dl} \times \vec{r})}{r^3}$$



→ Ampere Circuital law:

$$\oint B \cdot dl = \mu_0 I_{enclosed}$$

Application:

→ Magnetic Field due to a long straight current carrying wire;

$$B = \frac{\mu_0 I}{2\pi r}$$

→ Magnetic field induction of solenoid

$$B = \mu_0 n i$$

→ Properties of $\vec{B}$

1. $\oint B \cdot dl = \mu_0 I$

2. $\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$ or $\text{curl} \vec{B} = \mu_0 \vec{J}$

3. $\vec{\nabla} \cdot \vec{B} = 0$ or $\text{div } \vec{B} = 0$

These handwritten notes are of PHY-S102 taught to us by Prof. Prabal Pratap Singh, compiled and organized chapter-wise to help our juniors. We hope they make your prep a bit easier.

— **Saksham Nigam** and **Misbahul Hasan** (B.Tech. CSE(2024-28)