Q. Find the ratio electric force to gravitational force between
(i) proton and electron
(ii) proton and proton
(iii) electron and electron.

$\therefore \quad F_E = \dfrac{K q_1 q_2}{r^2} \quad ; \quad F_G = \dfrac{G m_1 m_2}{r^2}$

$\therefore \quad \dfrac{F_E}{F_G} = \dfrac{K q_1 q_2}{G m_1 m_2}$

(i) proton and electron

$\therefore \quad q_p = 1.6 \times 10^{-19} C$

$q_e = 1.6 \times 10^{-19} C$

$m_p = 1.67 \times 10^{-27} Kg$

$m_e = 9.1 \times 10^{-31} Kg$

$\therefore \quad \dfrac{F_E}{F_G} = \dfrac{K q_1 q_2}{G m_1 m_2} = \dfrac{9 \times 10^9 \times (1.6 \times 10^{-19})^2}{6.67 \times 10^{-11} \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19}}$

$\dfrac{F_E}{F_G} = 2.37 \times 10^{39}$

(ii) proton and proton

$\Rightarrow \quad \dfrac{F_E}{F_G} = \dfrac{9 \times 10^9 \times (1.6 \times 10^{-19})^2}{6.67 \times 10^{-11} \times (1.6 \times 10^{-27})^2}$

$\Rightarrow \quad \dfrac{F_E}{F_G} = 1.34 \times 10^{36}$

(iii) electron and electron

$\therefore \quad \dfrac{F_E}{F_G} = \dfrac{9 \times 10^9 \times (1.6 \times 10^{-19})^2}{6.67 \times 10^{-11} \times (9.1 \times 10^{-31})^2}$

$\Rightarrow \quad \dfrac{F_E}{F_G} = 4.17 \times 10^{42}$

Q. Two small insulated copper sphere A and B of some size have their centers 50 cm apart in air.

(i) Find the mutual force of repulsion if charge on each is 6.5×10^{-7}C.

(ii) What would be the force of repulsion if the charge in each sphere is doubled and distance between them be halved.

(iii) A third identical uncharged sphere C to be touched with 'A' and then with 'B' and finally removed away.

2

(i) $F = \dfrac{Kq_1q_2}{r^2} = \dfrac{9\times10^9\times(6.5\times10^{-7})^2}{(50\times10^{-2})^2} = 0.01521\ N$

(ii) $F' = \dfrac{K(2q_1)(2q_2)}{(r/2)^2} = 16F = 0.24336\ N$

(iii) $F = \dfrac{q_1q_2}{4\pi\epsilon_0 K r^2} = \dfrac{F}{K} = \dfrac{0.01521}{81} = 1.87\times10^{-4}\ N$

(iv) Final charge on $A = q/2$

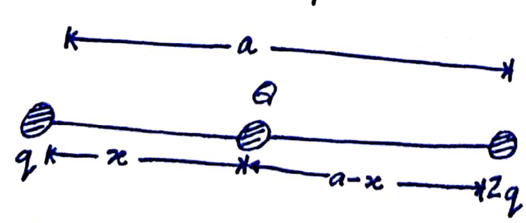
$B = 3q/4$

$\therefore\ F = \dfrac{K\frac{q_1}{2}\cdot\frac{3q_2}{4}}{r^2} = \dfrac{3}{8}\dfrac{Kq_1q_2}{r^2} = \dfrac{3}{8}\times0.01521 = 0.0057\ N$

Ans.

Q. Two point charge q and 2q are at a distance 'a' apart from each other in air. A third charge 'Q' is to be placed along the same line such that the net force acting on q and 2q is zero. Calculate the position of charge Q in terms of q and a.

$\therefore$



$\therefore$ F_{net} on $q = 0$

$\Rightarrow \dfrac{K\cancel{q}Q}{x^2} + \dfrac{2\cancel{K}q^{\cancel{2}}}{a^2} = 0$

$\therefore \dfrac{Q}{x^2} + \dfrac{2q}{a^2} = 0$ —— (1)

$Q = -\dfrac{2qx^2}{a^2}$

F_{net} on $2q = 0$

$\therefore \dfrac{\cancel{2}Q\cancel{q}}{(a-x)^2} + \dfrac{\cancel{2}q^{\cancel{2}}}{a^2} = 0$

$\dfrac{Q}{(a-x)^2} + \dfrac{q}{a^2} = 0$ —— (2)

$Q = -\dfrac{q}{a^2}\cdot(a-x)^2$

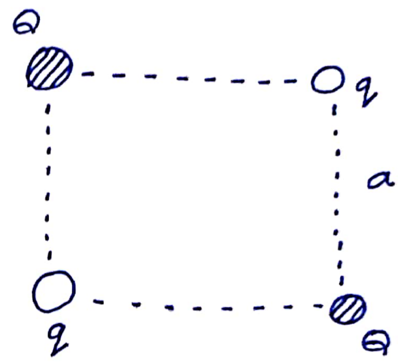
$$\frac{+2\cancel{q}x^2}{a^2} = +\frac{\cancel{q}}{a^2}(a-x)^2$$

$$\sqrt{2}\,\frac{x}{\cancel{x}} = \frac{a-x}{\cancel{x}}$$

$\therefore$ $\cancel{x\left(1+\frac{\sqrt{2}}{a}\right) = a}$ $\qquad a = x(\sqrt{2}+1)$

$\Rightarrow \cancel{x}$ $\qquad \therefore x = \dfrac{a}{\sqrt{2}+1}$. Ans.

Q. 2 equal point charges $Q = +2\mu C$ are placed at each of two opposite corner of a square at equal point charges q at each of the other two corridor what must be the value of q so that the resultant force on Q is zero.

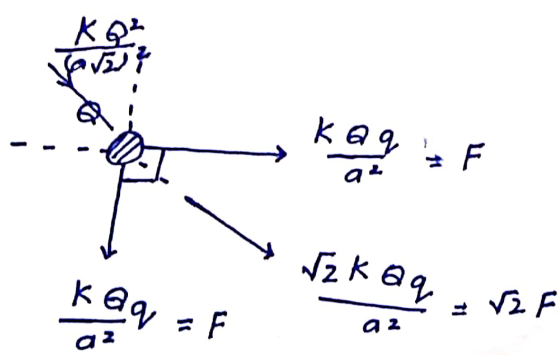


$\therefore$ F_{net} on $Q = 0$

Hence

$$\frac{K\cancel{Q}\cdot Q}{(a\sqrt{2})^2} + \frac{\sqrt{2}K\cancel{Q}q}{a^2} = 0$$

As,



$$\Rightarrow \frac{Q}{2\cancel{a^2}} + \frac{\sqrt{2}q}{\cancel{a^2}} = 0$$

$$q = -\frac{Q}{2\sqrt{2}}. \text{ Ans}$$

Q. An electron is seperated from a proton by a distance of $0.53\,Å$. Calculate the electric field at the location of electron.

$$E = \frac{Kq}{r^2}$$; As hydrogen nucleus only have one proton.

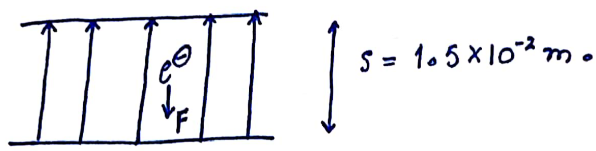
$\Rightarrow \quad q = q_p = 1.6\times10^{-19}C$

$$\therefore \quad E = \frac{9\times10^{9}\times1.6\times10^{-19}C}{(0.53\times10^{-10})^2} = 27.16\ N/C \text{ Ans.}$$

Q. A electron falls through a distance 1.5 cm in an uniform electric field of magnitude 2×10^4 N/C. The direction of the field is reversed and a proton falls through the same distance. Calculate the time of fall in each case ignoring gravity.

4

$\therefore$



$\therefore \quad F_{e\ominus} = qE = (1.6 \times 10^{-19})(2 \times 10^{4}) = 3.2 \times 10^{-15}\,N$

Hence, $a_{eff} = \dfrac{F}{m_e} = \dfrac{3.2 \times 10^{-15}\,N}{9.1 \times 10^{-31}\,kg} = 3.51 \times 10^{15}\,m/s^2.$

$\therefore$ Using eq of motion:

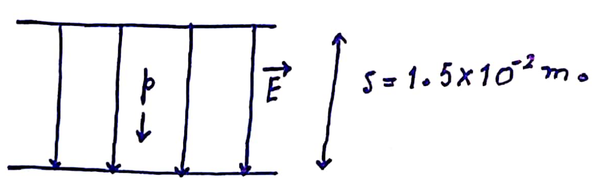
$u = 0$

$\therefore \quad s = ut + \frac{1}{2} a t^2$

$(1.5 \times 10^{-2}) = \frac{1}{2} \times 3.51 \times 10^{15} \times t_{fall}^2$

$\Rightarrow \quad t_{fall\ for\ e\ominus} = 2.92 \times 10^{-9}\ s$

Now,



$\therefore \quad F_p = (1.6 \times 10^{-19})(2 \times 10^{4}) = 3.2 \times 10^{-15}$

Hence,

$a_{eff} = \dfrac{3.2 \times 10^{-15}}{1.67 \times 10^{-27}} = 1.91 \times 10^{12}\ m/s^2$

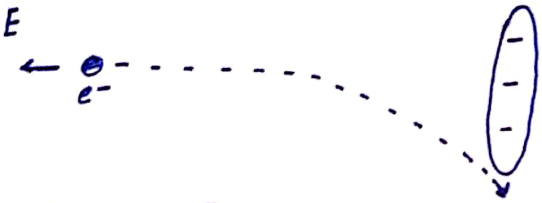
$\therefore \quad s = ut + \frac{1}{2} a t^2$

$(1.5 \times 10^{-2}) = \frac{1}{2} \times 1.91 \times 10^{12} \times t_{fall}^2$

$t_{fall}^2 = \dfrac{2 \times 1.5 \times 10^{-2}}{1.91 \times 10^{12}}$

$t_{fall} = 1.57 \times 10^{-14}\ s.$ Ans.

Q. From what distance should 100eV electron be fired towards a large metal plate having a surface charge density $2 \times 10^{-6}\ C\ m^{-2}$, so that it just fails to strike the plate.

∴ 

$$\Rightarrow \quad V_{e^{\ominus}} = E_{\text{due to plate}}$$

$$100 = \frac{\sigma}{\varepsilon_0} d$$

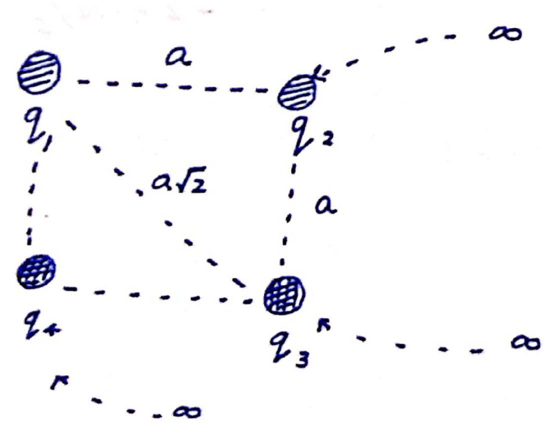
$$\therefore \quad d = \frac{100 \times 8.85 \times 10^{-12}}{2 \times 10^{-6}} = 4.425 \times 10^{-4}\ m.$$

Q. Four equal charge 'q' are brought from ∞ to the four corners A, B, C and D of a square of side 'a'. Compute the work required in bringing

(i) The first charge.
(ii) The second charge.
(iii) The third charge.
(iv) The fourth charge.
(v) What will be the electrostatic potential energy of the whole system.

$$\therefore \quad W_{\text{on } q_1} = 0$$

Now; $$W_{\text{for } q_2} = \frac{K q_1 q_2}{a}$$



$$W_{\text{for } q_3} = \frac{K q_1 q_3}{a\sqrt{2}} + \frac{K q_2 q_3}{a}$$

$$W_{\text{for } q_4} = \frac{K q_1 q_4}{a} + \frac{K q_2 q_4}{a\sqrt{2}} + \frac{K q_3 q_4}{a}$$

$$\therefore \quad U = \frac{K q_1 q_2}{a} + \frac{K q_1 q_3}{a\sqrt{2}} + \frac{K q_2 q_3}{a} + \frac{K q_1 q_4}{a} + \frac{K q_2 q_4}{a\sqrt{2}} + \frac{K q_3 q_4}{a} \text{. Ans.}$$

Q. An electron revolves around the nucleus of hydrogen atom in a circular orbit of radius 5×10^{-11} m. Calculate:

(i) Intensity of electric field of nucleus at the position of the electron.

(ii) Electrostatic potential energy of hydrogen nucleus and electron system.

(iii) An electric field is expressed in terms of potential: ⑥

$$V = 4x^2 + 3y^3 - 9z^2$$

Compute the intensity of the field at point (2, 3, 4).

(i) $\therefore \quad E = \frac{Kq_p}{r^2} = \frac{9\times10^9 \times 1.6\times10^{-19}}{5\times10^{-11}\times 5\times10^{-11}} =$

(ii) $U = \frac{Kq_1q_2}{r} = \frac{9\times10^9 \times (1.6\times10^{-19})^2}{5\times10^{-11}} =$

(iii) $E = -\nabla V = -\,\text{grad}(V)$

$\begin{array}{r} 18 \\ 4 \\ \hline 72 \end{array}$

$\therefore \quad V = 4x^2 + 3y^3 - 9z^2$

$\nabla V = 8x\hat{i} + 9y^2\hat{j} - 18z\hat{k}$

$\nabla V\Big|_{(2,3,4)} = 16 + 9(9) - 18(4) = 16\hat{i} + 81\hat{j} - 72\hat{k}$

$\therefore \quad \nabla V = 25$

$\Rightarrow |\vec{E}| = |\nabla V| = \sqrt{16^2+81^2+72^2} = 109.8\ N/c$ Ans.

Q. A spherically symmetric charge distribution of radius R is characterised by the charge distribution.

$$\rho(r) = \begin{cases} \rho_0\left(1 - \frac{r^2}{R^2}\right) & \text{for } r \leq R \\ 0 & , r > R \end{cases}$$

Calculate the 1. Total amount of charge

2. Electric field strength at a point P of distance r from the

(i) inside and

(ii) outside

the charge distribution.

(iii) The value of r from which electric field is maximum.

(i) $dq = \rho_0 \, dV$

$$\Rightarrow \int dq = \int_0^R \rho_0 \left(1 - \frac{r^2}{R^2}\right) \cdot 4\pi x^2 \, dx$$

$$= \rho_0 \, 4\pi \int_0^R r^2 \, dr - \frac{r^4}{R^2} \, dr$$

$V =$

$$Q = 4\pi \rho_0 \left[\frac{R^3}{3} - \frac{R^5}{5R^2} \right]$$

$$= 4\pi \rho_0 \left[\frac{5R^3 - 3R^3}{15} \right]$$

$$Q = \frac{8 \; 4\pi R^3 \rho_0}{15} \; C$$ Ans.

(ii) According to Gauss Law

(i) inside $\int E \cdot dA = \frac{Q}{\epsilon_0}$

$$\therefore \quad Q_{in} = \int_0^r \rho_0 \left(1 - \frac{r^2}{R^2}\right) \cdot 4\pi r^2 \, dr$$

$$Q_{in} = 4\pi\rho_0 \left[\left[\frac{r^3}{3}\right]_0^r - \frac{1}{R^2} \cdot \left[\frac{r^5}{5}\right]_0^r \right]$$

$$E \, \cancel{4\pi} r^2 = \frac{\cancel{4\pi}\rho_0}{\epsilon_0} \left[\frac{r^3}{3} - \frac{r^5}{5R^2}\right]$$

$$\therefore \quad \vec{E} = \frac{\rho_0}{\epsilon_0}\left[\frac{r}{3} - \frac{r^3}{5R^2}\right] \quad \text{Ans.}$$

$\Rightarrow$ (ii) outside $\int E \cdot dA = \frac{Q}{\epsilon_0}$

$$\therefore \quad Q_{in} = \int_0^R \rho_0 \left(1 - \frac{r^2}{R^2}\right) \cdot 4\pi r^2 \cdot dr$$

$$Q_{in} = \frac{8\pi\rho_0 R^3}{15}$$

$$\therefore \quad \vec{E} = \frac{2\rho_0 R^3}{\epsilon_0 15 r^2} \quad \text{Ans.}$$

$\therefore$ (iii) $$\frac{dE}{dr} = \frac{\rho_0}{\epsilon_0}\left[\frac{1}{3} - \frac{3r^2}{5R^2}\right] = 0$$

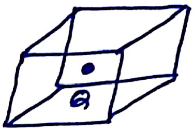
$$\Rightarrow \quad \frac{1}{3} - \frac{3r^2}{5R^2} = 0$$

$$r = \frac{5R^2}{9}$$

$$r = \frac{R\sqrt{5}}{3} \quad \text{Ans.}$$

Q. A point charge of 10^{-6} C is situated at the center of cube of 1m side. Calculate the electric flux through it's surface. (9)

∴ According to Gauss's Law



∴ $\phi_E = \frac{Q}{\epsilon_0}$

∴ ϕ_E through cube $= \frac{10^{-6}}{\epsilon_0}$

ϕ_E through surface of cube $= \frac{10^{-6}}{6\epsilon_0} = 18832.39 \frac{Nm^2}{C}$ Ans.

$= 1.88 \times 10^5 \frac{Nm^2}{C}$

Q. In a region of space the electric field is given by $\vec{E} = 8\hat{i} + 4\hat{j} + \hat{k}$. Calculate the electric flux through the surface $100\hat{k}$.

∴ $\phi_E = \int \vec{E} \cdot d\vec{A} = (8\hat{i} + 4\hat{j} + \hat{k}) \cdot (100\hat{k}) = 100 \frac{Nm^2}{C}$ Ans.

Q. A polyethene rubbed with wool is found to have -3×10^{7} C.

(a) Calculate no. of electrons transferred also state from which the transfer of $e^{\ominus}$ took place.

(b) Is there a transfer of mass from wool to polyethene.

∴ (a) ⇒ $q = ne$

$$-3 \times 10^{7} = n \times (-1.6 \times 10^{-19})$$

$$n = 1.875 \times 10^{26} \quad \text{Ans}$$

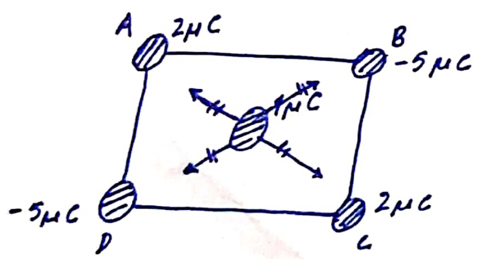
∴ As polyethene have negative charge.

⇒ $e^{\ominus}$s transferred from wool to polyethene.

(b). Yes, transfer of mass also takes places.

Q. Four point charges, $Q_A = 2\mu C$, $Q_B = -5\mu C$, $Q_C = 2\mu C$ and $Q_D = -5\mu C$ are located at the corner of square ABCD of side 10 cm. What is the force on a charge of $1\mu C$, placed at the centre.

∴



$\vec{F}_{net} = 0$ Ans.

Q. Charge on an electron is 1.6×10^{-19} and it's mass is 9×10^{-31} kg. Calculate the acceleration of an electron in an Electric field of 9×10^{5} N/C.

∴

$$F = qE$$

$$F = 1.6 \times 10^{-19} \times 9 \times 10^{5}$$

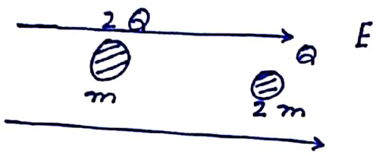
$$F = 1.44 \times 10^{-13} \text{ N}$$

∴

$$a = \frac{F}{m} = \frac{1.44 \times 10^{-13}}{9 \times 10^{-31}} = 1.6 \times 10^{17} \text{ m/s}^2 \text{ . Ans.}$$

Q. Two charged particle of massess 'm' and '2m' have charges +2Q and +Q respectively, they are kept in uniform electric field and allowed to move for the same time. Find the ratio of thier kinetic energies.

$\therefore$



$\therefore \quad a_1 = \dfrac{2QE}{m}, \quad a_2 = \dfrac{QE}{2m}$

$\therefore \quad \dfrac{KE_1}{KE_2} = \dfrac{m}{2m}\left(\dfrac{V_1}{V_2}\right)^2$

$\therefore \quad V_1 = a_1 t$

$V_2 = a_2 t$

$\therefore \quad \dfrac{KE_1}{KE_2} = \dfrac{1}{2}\left[\dfrac{2\cancel{QE}\cdot 2\cancel{m}}{\cancel{m}\cdot\cancel{QE}}\right]^2 = \dfrac{4\times\cancel{4}^{2}}{\cancel{2}} = 8$ Ans.

$\vec{r} = |r| \cdot \hat{r}$

Q. A point charge of 10^{-9}C is placed at the origin. Find the potential V_A (1,2,2) and B (4,4,2).

$\therefore \quad V_A = \dfrac{kq_1}{|r_1|}\cdot\dfrac{\vec{r_1}}{|r_1|} = \dfrac{kq\cdot\vec{r_1}}{|r_1|^2}$

$\vec{r_1} = \hat{i} + 2\hat{j} + 2\hat{k}$

$|\vec{r_1}| = \sqrt{1+4+4} = 3$

$V_A = \dfrac{\cancel{9}\times\cancel{10^9}\times\cancel{10^{-9}}\times(\hat{i}+2\hat{j}+2\hat{k})}{\cancel{9}}$

16+16+4

$\therefore \quad |V_A| = \sqrt{(\hat{i}+2\hat{j}+2\hat{k})} = 3V$ Ans.

$\therefore \quad V_B = \dfrac{kq}{|r_B|} = \dfrac{\cancel{9}^{3}\times\cancel{10^{-9}}\times\cancel{10^{+9}}}{\cancel{6}2} = 1.5V$ ans.

Q. To what potential function at a point is given by $V = x(3y^2 - x^2 + z)$ V. Find the components of electrostatic field at that point.

$\therefore \quad E = -\nabla V$

$\nabla = \dfrac{\partial}{\partial x}\hat{i} + \dfrac{\partial}{\partial y}\hat{j} + \dfrac{\partial}{\partial z}\hat{k}$

$\therefore \quad E = -\left[\dfrac{\partial}{\partial x}(3y^2x - x^3 + zx)\hat{i} + \dfrac{\partial}{\partial y}(3y^2x - x^3 + zx)\hat{j} + \dfrac{\partial}{\partial x}(3y^2x - x^3 + zx)\hat{k}\right]$

$$E = -\left[(3y^2 - 3x^2 + z)\hat{i} + 6x\hat{j} + x\hat{k}\right]$$

$$\therefore \vec{E} = (3x^2 - 3y^2 + z)\hat{i} - 6x\hat{j} - x\hat{k}$$

$$\Rightarrow E_x = 3x^2 - 3y^2 + z$$

$$E_y = -6x \qquad \underline{\underline{Ans}}.$$

$$E_z = -x$$

Q. To what potential must be charge insulated sphere 22.1 cm radius so that it's surface density maybe $1\ C/m^2$.

$$\therefore \sigma = \frac{Q}{A}$$

$$\Rightarrow Q = \sigma . A$$

$$= 1 \times 4\pi (22.1 \times 10^{-2})^2$$

$$Q = 0.6137\ C$$

$$\therefore V = \frac{KQ}{r^2}$$

$$= \frac{1}{\cancel{4\pi}\epsilon_0} \times \frac{\sigma \times \cancel{4\pi} \cancel{r^2}}{\cancel{r^2}}$$

$$V = \frac{\sigma}{\epsilon_0} = \frac{1}{8.85 \times 10^{-12}} = 1.12 \times 10^{11}\ V \ \underline{\underline{Ans}}.$$

Q. Show that $V = \frac{A}{r} + B$ satisfy Laplace equation where A and B are constant and r is the magnitude of position vector.

(13)

$\therefore$ $\boxed{\nabla^2 V = 0} \rightarrow$ Laplace Equation

$\Rightarrow$ $\text{div}(\text{grad } V) = 0$

$\therefore$ $V = \frac{A}{r} + B$

$$r = \sqrt{x^2 + y^2 + z^2}$$

$\therefore$ $$\text{grad } V = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)\left(\frac{A}{\sqrt{x^2+y^2+z^2}} + B\right)$$

$\therefore$ $$A \cdot \frac{\partial}{\partial x}(x^2+y^2+z^2)^{-1/2}\hat{i} + A\frac{\partial}{\partial y}(x^2+y^2+z^2)^{-1/2}\hat{j} + \frac{A}{1}\frac{\partial}{\partial z}(x^2+y^2+z^2)^{-1/2}\hat{k}$$

$\therefore$ $$\text{grad } V = \frac{-1A}{\cancel{2}}\left(\frac{\cancel{2}x\,\hat{i}}{(x^2+y^2+z^2)^{3/2}} + \frac{\cancel{2}y}{(x^2+y^2+z^2)^{3/2}} + \frac{\cancel{2}z}{(x^2+y^2+z^2)^{3/2}}\right) + 0$$

$$\text{grad } V = -A\left(\frac{x\hat{i} + y\hat{j} + z\hat{k}}{(x^2+y^2+z^2)^{3/2}}\right)$$

$$\text{div}(\text{grad } V) = -A\left(\frac{(x^2+y^2+z^2)^{3/2} - \frac{3x}{2}(x^2+y^2+z^2)^{1/2}\cdot(2x)}{(x^2+y^2+z^2)^3} + \frac{(x^2+y^2+z^2)^{3/2} - \frac{3}{2}y(x^2+y^2+z^2)^{1/2}\cdot(2y)}{(x^2+y^2+z^2)^3} + \frac{(x^2+y^2+z^2)^{3/2} - \frac{3}{2}z(x^2+y^2+z^2)^{1/2}\cdot(2z)}{(x^2+y^2+z^2)^3}\right)$$

$$\nabla^2 V = -A\left(\frac{r^2 - 3x^2}{r^5} + \frac{r^2 - 3y^2}{r^5} + \frac{r^2 - 3z^2}{r^5}\right) = \frac{3r^2 - 3(x^2+y^2+z^2)}{r^5}$$

$$= 0$$

Hence, $\boxed{\nabla^2 V = 0}$

These handwritten notes are of PHY-S102 taught to us by Prof. Prabal Pratap Singh, compiled and organized chapter-wise to help our juniors. We hope they make your prep a bit easier.

— **Saksham Nigam** and **Misbahul Hasan** (B.Tech. CSE(2024-28)