Interference of Light

Superimposition of two waves:

Let us consider two monochromatic waves of same frequency (ω), amplitudes a_1, a_2 and phase difference ϕ.

$$\therefore \quad y_1 = a_1 \sin\omega t \quad \text{--- (1)}$$

$$y_2 = a_2 \sin(\omega t + \delta) \quad \text{--- (2)}$$

Resultant displacement by principle of superimposition:

$$y = y_1 + y_2$$

$$y = a_1 \sin\omega t + a_2 \sin(\omega t + \delta)$$

$$y = a_1 \sin\omega t + a_2 \sin\omega t \cos\delta + a_2 \cos\omega t \sin\delta$$

$$y = (a_1 + a_2\cos\delta)\sin\omega t + (a_2 \sin\delta)\cos\omega t.$$

Let $\quad a_1 + a_2\cos\delta = A\cos\phi \quad \text{--- (3)}$

$\quad a_2 \sin\delta = A\sin\phi \quad \text{--- (4)}$

$$\therefore \quad y = A\cos\phi \sin\omega t + A\sin\phi\cos\omega t$$

$$y = A\sin(\omega t + \phi)$$

Squaring and adding (3) and (4)

$$\therefore \quad (a_1 + a_2\cos\delta)^2 + (a_2\sin\delta)^2 = A^2\cos^2\phi + A^2\sin^2\phi$$

$$a_1^2 + a_2^2\cos^2\delta + 2a_1a_2\cos\delta + a_2^2\sin^2\delta = A^2$$

$$\Rightarrow \quad A^2 = a_1^2 + a_2^2 + 2a_1a_2\cos\delta$$

$$A_R = \sqrt{a_1^2 + a_2^2 + 2a_1a_2\cos\phi}$$

Resultant Amplitude

And, $\quad$ Intensity $\propto$ Amplitude^2

$$I \propto A^2$$

$\therefore$ We get, $\quad I_R = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos\phi$

* If $I_1 = I_2 = I$

$\therefore \quad I_R = I + I + 2I\cos\phi$

$= 2I(1+\cos\phi)$

$I_R = 4I\cos^2\frac{\phi}{2}$ ✓

Conditions for:

1. For constructive interference (Maxima)

$\phi = 2n\pi \quad , \quad n = 0, 1, 2, \ldots$

$\therefore \quad \cos\phi = 1$

$\Rightarrow \quad I_R = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2} = (\sqrt{I_1}+\sqrt{I_2})^2.$

If $I_1 = I_2 = I$

$\Rightarrow \quad I_{Max} = 4I.$ /

and

$A_R^2 = A_1^2 + A_2^2 + 2A_1A_2 = (A_1+A_2)^2$

$\Rightarrow \quad A_R = A_1 + A_2.$

2. For destructive interference (Minima)

$\phi = (2n-1)\pi \quad , \quad n = 1, 2, 3, \ldots$

$\cos\phi = -1.$

$\therefore \quad I_R = I_1 + I_2 - 2\sqrt{I_1}\sqrt{I_2}$

If $I_1 = I_2 = I$

$\Rightarrow \quad I_{Min} = 4I\cos^2\frac{(2n-1)\pi}{2} = 0$

and $\quad A_R^2 = A_1 + A_2 + (-2A_1A_2) = (A_1 - A_2)^2$

$A_R = A_1 - A_2$

→ Path difference for Constructive Interference

$\therefore \quad \phi = 2\pi n \quad , \quad n = 0, 1, 2, \ldots$

and $\quad \Delta = \frac{\lambda}{2\pi} \times \phi$

Path difference

$\therefore \quad \Delta = \frac{\lambda}{2\pi} \times 2\pi n = n\lambda$

$\boxed{\Delta = n\lambda}$ → integral multiple of wavelength. (λ)

→ Path difference for Destructive Interference

$\phi = (2n-1)\pi \quad , \quad n = 1, 2, 3, \ldots$

and $\quad \Delta = \frac{\lambda}{2\pi} \times \phi$

$\therefore \quad \Delta = \frac{\lambda}{2\cancel{\pi}} \times (2n-1)\cancel{\pi} = \frac{(2n-1)\lambda}{2}$

$\boxed{\Delta = \frac{(2n-1)\lambda}{2}}$ → odd integral multiple of $\lambda/2$.

Q1. If in an interference pattern, the ratio between maximum and minimum intensity is 36:1. Find the ratio between amplitude and intensities of the two interfering waves.

Let us consider A_1 and A_2 be the amplitudes of the interfering waves and I_1 and I_2 be the intensities.

$$\Rightarrow \quad \frac{I_{Max}}{I_{Min}} = \frac{36}{1}$$

and
$$\frac{(A_1 + A_2)^2}{(A_1 - A_2)^2} = \frac{36}{1}$$

$$\therefore \quad \frac{A_1 + A_2}{A_1 - A_2} = \frac{6}{1}$$

$$\frac{\cancel{A_2}\left(\frac{A_1}{A_2} + 1\right)}{\cancel{A_2}\left(\frac{A_1}{A_2} - 1\right)} = \frac{6}{1}$$

$$\frac{A_1}{A_2} + 1 = 6\frac{A_1}{A_2} - 6$$

$$5\frac{A_1}{A_2} = 7$$

$$\frac{A_1}{A_2} = \frac{7}{5}.$$

and
$$I \propto A^2$$

$$\therefore \quad \frac{I_1}{I_2} = \left(\frac{A_1}{A_2}\right)^2 = \frac{49}{25}. \text{ Ans.}$$

Q2. Two coherant sources where intensity ratio is 100:1, produce interference fringes. Find the ratio of maximum intensity in fringe system.

$\therefore \quad \frac{I_1}{I_2} = \frac{100}{1}$

Intensity

As; $\hookrightarrow I \propto A^2 \longrightarrow$ Amplitude

$\Rightarrow \quad \frac{A_1^2}{A_2^2} = \frac{100}{1} \Rightarrow \frac{A_1}{A_2} = 10$

$\therefore \quad \frac{I_{Max}}{I_{Min}} = \frac{(A_1+A_2)^2}{(A_1-A_2)^2} = \frac{\left(\frac{A_1}{A_2}+1\right)^2}{\left(\frac{A_1}{A_2}-1\right)^2} = \frac{(10+1)^2}{(10-1)^2} = \frac{11^2}{9^2}$

$\Rightarrow \quad \frac{I_{Max}}{I_{Min}} = \frac{121}{81}$. Ans.

Q3. Two coherant beams of wavelength 5000Å reaching a point would individually produce intensities 1.44 and 4.0 unit respectively. If they reach there together the intensity is 0.90 unit. Calculate the lowest phase difference with which the beam reach that point.

$\therefore \quad I_R = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos\phi$

$0.90 = 1.44 + 4 + 2(\sqrt{1.44})(\sqrt{4})\cos\phi$

$\cos\phi = \frac{0.90 - 1.44 - 4}{2 \times 1.2 \times 2}$

$\cos\phi = -0.945$

$\cos(180° - \phi) = 0.945$

$180° - \phi = \cos^{-1}(0.945)$

$180° - \phi = 19.09$

$\phi = 180° - 19.09 = 161.91°$ Ans.

Q. Two coherent sources of intensity ratios "α". Prove that in the interference pattern;

$$\frac{I_{Max} - I_{Min}}{I_{Max} + I_{Min}} = \frac{2\sqrt{\alpha}}{1+\alpha}$$

$$I_{Max} = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}$$
$$I_{Min} = I_1 + I_2 - 2\sqrt{I_1}\sqrt{I_2} \quad , \quad \alpha = \frac{I_1}{I_2}$$

$$\therefore \quad \frac{I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2} - (I_1 + I_2 - 2\sqrt{I_1}\sqrt{I_2}}{I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2} + I_1 + I_2 - 2\sqrt{I_1}\sqrt{I_2}}$$

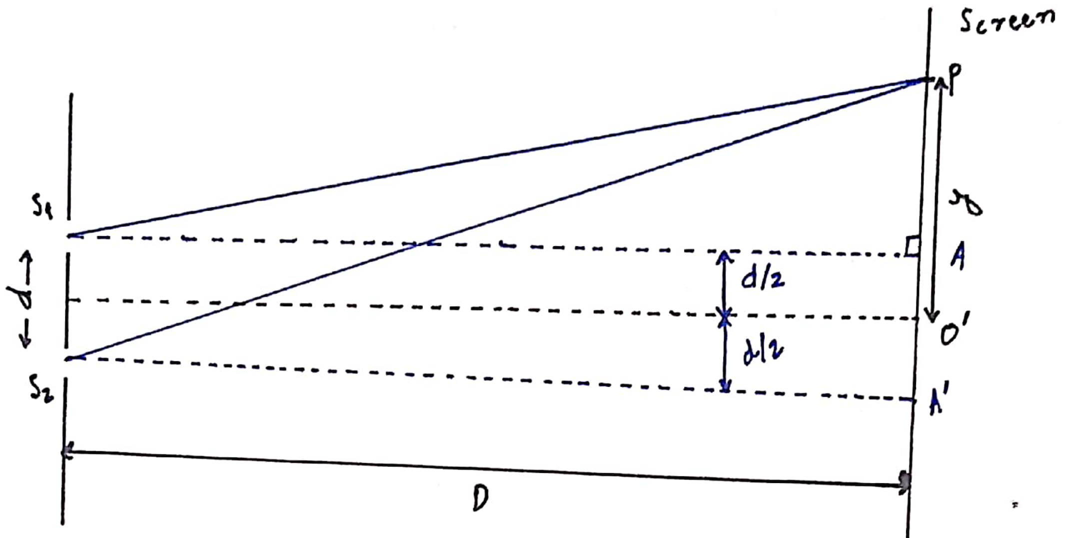
$$= \frac{4\sqrt{I_1}\sqrt{I_2}}{2(I_1 + I_2)} = \frac{2\sqrt{I_1}\sqrt{I_2}}{I_2\left(\frac{I_1}{I_2}+1\right)}$$

$$= \frac{2\sqrt{I_1/I_2}}{\frac{I_1}{I_2}+1} = \frac{2\sqrt{\alpha}}{1+\alpha}.$$

Hence Proved.

Young's Double Slit Experiment

→ Expression for positions of maxima and minima on screen in YDSE. (↱ distance from midpoint of screen.)



∴ From ΔS_1PA

$$(S_1P)^2 = D^2 + (y - d/2)^2$$

$$(S_1P)^2 = D^2\left(1 + \frac{1}{D^2}(y - d/2)^2\right)$$

$$S_1P = D\left(1 + \underbrace{\frac{1}{D^2}(y - d/2)^2}_{\text{This term is very small}}\right)^{1/2}$$

∴ Using Binomial Theorem :

$$S_1P = D\left[1 + \frac{1}{2D^2}(y - d/2)^2\right]$$

$$S_1P = D + \frac{1}{2D}(y - d/2)^2 \text{ . Ans.}$$

Similarly :

$$S_2P = D + \frac{1}{2D}(y + d/2)^2.$$

$$\Delta = S_2P - S_1P = \frac{1}{2D}(y + d/2)^2 - \frac{1}{2D}(y - d/2)^2$$

$$\Delta = \frac{1}{2D}\left[\cancel{2}y\frac{d}{\cancel{2}} + \cancel{2}y\frac{d}{\cancel{2}}\right] = \frac{yd}{D}$$

$$\Rightarrow \quad \Delta = \frac{yd}{D}$$

For maxima;

$$\Delta = n\lambda \quad , \quad n = 0, 1, 2, \ldots$$

$$\therefore \quad n\lambda = \frac{y_n d}{D}$$

$$\Rightarrow \quad y_n = \frac{n\lambda D}{d}$$ → Position of bright fringe

For minima;

$$\Delta = (2n-1)\frac{\lambda}{2}, \quad n = 1, 2, 3, \ldots$$

$$(2n-1)\frac{\lambda}{2} = \frac{y_n d}{D}$$

$$\Rightarrow \quad y_n = \frac{(2n-1)\lambda D}{2d}.$$ → Position of dark fringe.

→ Fringe Width (β): Distance between any two consecutive dark or bright fringe.

$$\therefore \quad \beta = y_{2Bright} - y_{1Bright}$$

$$= \frac{2\lambda D}{d} - \frac{\lambda D}{d}$$

$$\therefore \quad \beta = \frac{\lambda D}{d}.$$

Angular Fringe Width;

$$\alpha = \beta/D = \frac{\lambda}{d}.$$

Q. Calculate the seperation between consecutive bright and dark fringes.

$$y_{bright} = \frac{n\lambda D}{d}$$

$$y_{dark} = \frac{(2n-1)\lambda D}{2d}$$

$\therefore$ y seperation between consecutive bright $= y_{n+1} - y_n = \frac{(n+1)\lambda D}{d} - \frac{n\lambda D}{d} = \frac{\lambda D}{d}$

and,

y seperation between consecutive dark $= y_{n+1} - y_n = \frac{(2(n+1)-1)\lambda D}{2d} - \frac{2(n+1)\lambda D}{2d} = \frac{\lambda D}{d}$

Q. In a two slit interference pattern at a point we observe 10^{th} order maxima for $\lambda = 7000\,\mathring{A}$. What order will be visible if the source light is replaced by light of wavelength $5000\,\mathring{A}$.

$\therefore$

$$y_n = \frac{n\lambda D}{d} \quad \text{(Because maxima, i.e. bright fringe is forming)}$$

$$y = \frac{10 \times 7000 \times D}{d}$$

then light is replaced by $\lambda = 5000\,\mathring{A}$

$$\Rightarrow \quad y_n = \frac{n\lambda D}{d}$$

$$y = \frac{n \times 5000 \times D}{d}$$

$$\Rightarrow \quad \frac{\overset{2}{\cancel{10}} \times 7\cancel{000} \times \cancel{D}}{\cancel{d}} = \frac{n \times \cancel{5}\cancel{000} \times \cancel{D}}{\cancel{d}}$$

$$\Rightarrow \quad n = 14$$

Hence, 14^{th} order will be visible.

Q. A double slit of seperation of 1.5 mm is illuminated by white light (between 4000 - 8000Å) on a screen 120 cm away coloured interference pattern is formed to start if a pinhole is made on this screen at a distance of 3 mm from the central white fringe what wavelength will be absent in the transmitted light.

$$\therefore \quad y = \frac{n\lambda D}{d}$$

$$\therefore \quad \lambda = \frac{yd}{nD} = \frac{3\times10^{-3}\times 1.5\times10^{-3}}{n\times 120\times10^{-2}}$$

$$\lambda = \frac{4.5\times10^{-6}}{n\times120\times10^{-2}} \times \frac{10^{-10}}{10^{-10}}$$

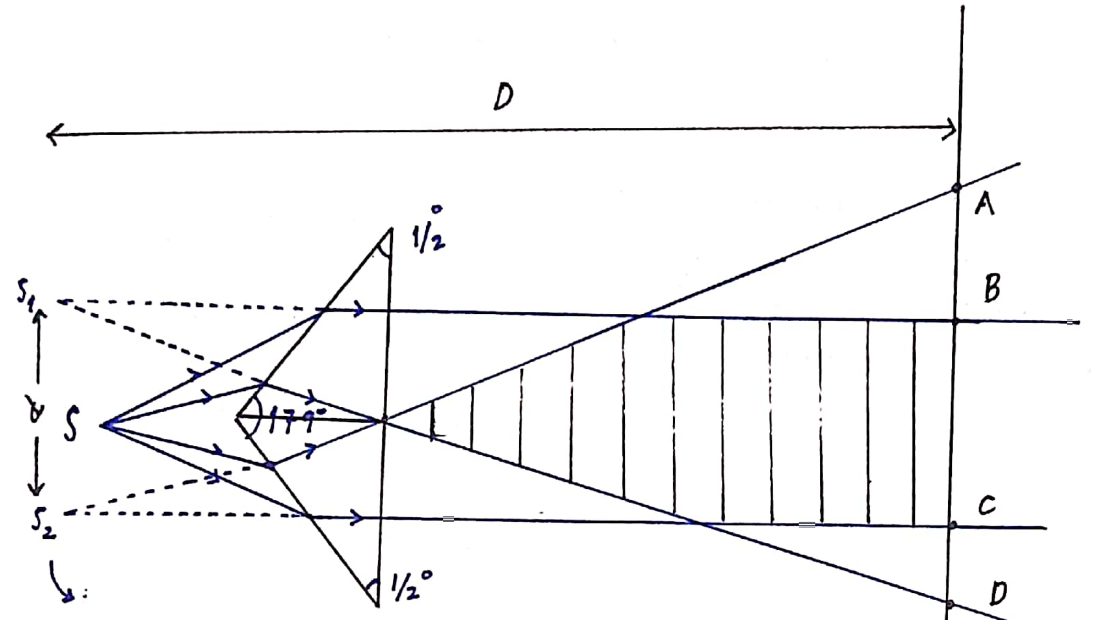
$$\lambda = \frac{37500}{n}\,\text{Å}$$

$\therefore$ At $n = 10$

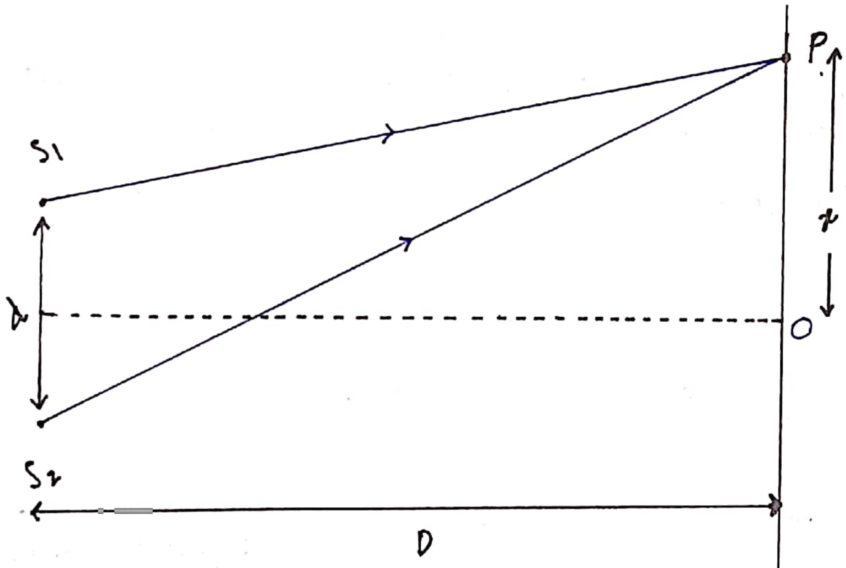
$\lambda = 3750$ Å $\rightarrow$ is ~~the~~ the wavelength that will be absent.

As; $\lambda = 3750$ Å lies outside visible spectrum (4000-8000Å)

→ Fresnel's Biprism



Secondary source : which are virtual images of S.



$\therefore \quad \Delta = S_2P - S_1P$

$\Delta = \dfrac{xd}{D}$ [same derivation as in YDSE]

$\therefore$ For maxima :

$\Delta = n\lambda \quad (n = 0, 1, 2, \ldots)$

$\therefore \quad n\lambda = \dfrac{xd}{D}$

$x = \dfrac{n\lambda D}{d}$.

For minima

$\Delta = (2n-1)\lambda \quad (n = 1, 2, 3, \ldots)$

$(2n-1)\lambda = \dfrac{xd}{D}$

$x = \dfrac{(2n-1)\lambda D}{d}$.

Fringe width:

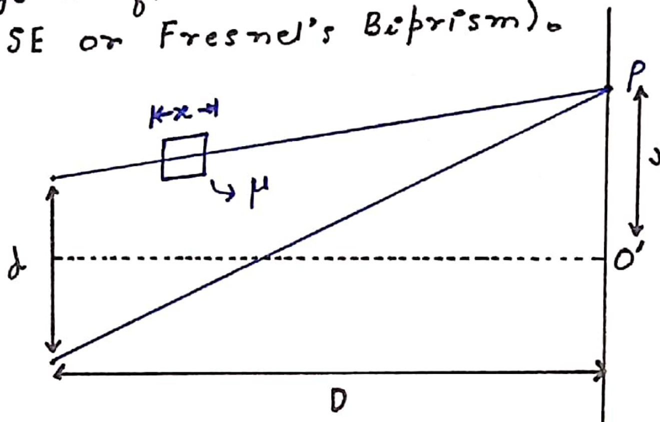
$$\beta = \frac{\lambda D}{d}.$$

Seperation between two virtual sources (d):

$$d = 2x(\mu-1)\theta$$

$d \rightarrow$ distance of seperation between virtual sources
$x \rightarrow$ distance between biprism and slit (source)
$\mu \rightarrow$ refractive index of biprism
$\theta \rightarrow$ angle of inclination.

→ Fringe Shift due to glass slab in path of wave.
(YDSE or Fresnel's Biprism).



Normally,

$$y = \frac{\Delta D}{d}.$$

But,
On introducing a glass plate of thickness x in the path of a ray, the extrapath difference introduced is

$$\Delta' = (\mu-1)x$$

Let y' be the new position of same fringe:

$$\Rightarrow \quad y' = \frac{D}{d}(\Delta+\Delta') = \frac{D}{d}(\Delta+(\mu-1)x)$$

∴ shift in fringes:

$$\delta = y'-y = \frac{(\mu-1)xD}{d}.$$

$\mu \rightarrow$ refractive index of slab.

Q. A biprism is placed at a distance of 5 cm infront of a narrow slit illuminated by sodium light ($\lambda = 5890\times10^{-8}$ cm) and the distance between the virtual sources is found to be 0.05 cm. Find the width of the fringe observed in an eyepiece placed at a distance 75 cm from the biprism.

$\therefore$ $\lambda = 5890\times10^{-8}$ cm

$d = 0.05$ cm

$D = 75$ cm

$$\Rightarrow \quad \beta = \frac{\lambda D}{d} = \frac{5890\times10^{-8}\times 75}{0.05} = 0.088 \text{ cm ans.}$$

Q. The inclined faces of biprism of glass ($\mu = 1.5$) make an angle of 2° with the base. the slits is at 10 cm from the biprism and is illuminated by light of wavelength 5500 Å. Find the fringe width at a distance of 1 m from the prism.

$$\therefore \quad d = 2x(\mu-1)\theta \longrightarrow \text{in radians}$$

$$1^\circ = \frac{\pi}{180^\circ}$$

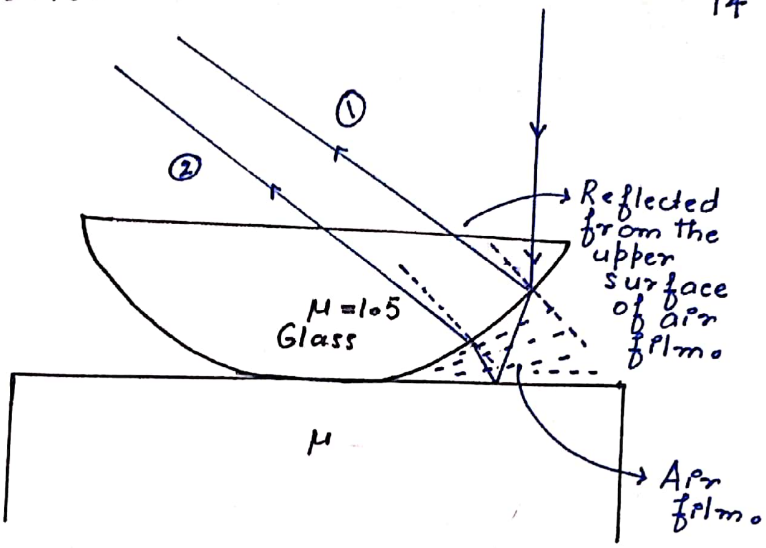
$$2^\circ = \frac{\pi}{90^\circ} = 0.0349$$

$$\Rightarrow \quad d = 2\times10\times(1.5-1)\times 0.0349$$

$$d = 0.349 \text{ cm}$$

$$\therefore \quad \beta = \frac{\lambda D}{d} = \frac{5500\times10^{-}\times100}{0.349} = 1.576\times10^{-2} \text{ cm Ans.}$$

Newton's Ring Experiment



∴ Path difference with thin film;

$$\Delta = 2\mu t \cos r + \underbrace{\frac{\lambda}{2}}$$

Path difference due to wave going to denser from rarer medium

$\mu \rightarrow$ refractive index of glass.

$t \rightarrow$ thickness at point of reflection

$r \rightarrow$ angle of refraction.

Note:

$\Delta = 2\mu t \cos(r+\theta) + \lambda/2$, here θ is angle of inclination at point of reflection but as radius of curvature is large, $\theta \approx 0$.

For normal incident light $r = 0$ $(\cos r = 1)$

$$\Rightarrow \quad \Delta = 2\mu t + \lambda/2. \quad \text{—} \ (1)$$

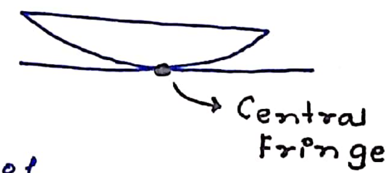
Path difference between rays (1) and (2)

Central Fringe: It is at point of contact.

$\Rightarrow \quad t = 0$

$\Delta = 2\mu(0) + \lambda/2$

also, $\mu = 1$ for air.



$\Delta = \frac{\lambda}{2}$, which is condition of destructive interference.

∴ The central fringe is <u>dark fringe.</u>

→ For maxima;

$\Delta = n\lambda \quad , \quad n = 1, 2, 3, \ldots$

$2\mu t + \frac{\lambda}{2} = n\lambda$

$\mu = 1$ (for air)

$2t + \frac{\lambda}{2} = n\lambda$

$\therefore \quad 2t = n\lambda - \frac{\lambda}{2}$

$2t = (2n-1)\frac{\lambda}{2}$ (For maxima)

$$\boxed{t = (2n-1)\frac{\lambda}{4}.}$$

For minima;

$$\Delta = (2n+1)\frac{\lambda}{2}, \quad n = 1, 2, 3$$

$$\cancel{2}t + \cancel{\frac{\lambda}{2}} = \cancel{2}n\frac{\lambda}{2} + \cancel{\frac{\lambda}{2}}$$

$$\therefore \boxed{2t = n\lambda}$$

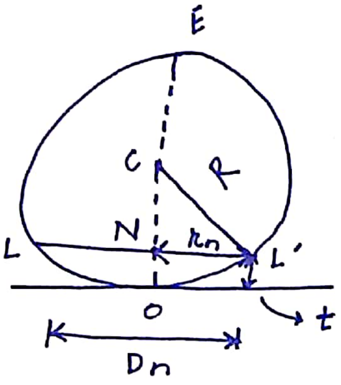
$$\boxed{t = \frac{n\lambda}{2}}$$

Here, bright or dark fringes of any order n will occur for a constant value of ~~thickness~~ of air film "t". Since 't' remains constant along a circle, with center at point of contact.

∴ Fringes are circular in shape (concentric circles).

⇒ Diameter of bright fringes;

D_n = diameter of n^{th} ring
r_n = radius of n^{th} ring at point P.
t = thickness of air film at P



∴ From circle's geometry

$$PN^2 = ON \times NE$$

$$r_n^2 = t \times (2R - t)$$

$$r_n^2 = 2Rt - t^2$$

As $t \to$ very small
$t^2 \to 0$

$$\therefore r_n = \sqrt{2Rt}$$

or $\boxed{2t = \frac{r_n^2}{R}}$

and from condition of bright fringe;

$$2t = (2n-1)\frac{\lambda}{2}$$

$$\frac{r_n^2}{R} = (2n-1)\frac{\lambda}{2}$$

$$r_n^2 = (2n-1)\frac{\lambda R}{2}$$

Putting $r_n = \frac{D_n}{2}$

$$\therefore D_n^2 = 2(2n-1)\lambda R$$

$$\boxed{D_n = \sqrt{2\lambda R(2n-1)}}$$

$(D_n \propto \sqrt{2n-1})$

$\therefore$ Diameters of bright fringes are in ratio of square roots of odd natural number

$$D_1 : D_2 : D_3 : \dots = \sqrt{1} : \sqrt{3} : \sqrt{7} \dots$$

$\Rightarrow$ Diameter of dark fringes

$$\therefore \quad 2t = \frac{r_n^2}{R}$$

and from condition of dark fringes

$$2t = n\lambda$$

$$\Rightarrow \quad n\lambda = \frac{r_n^2}{R}$$

$$\therefore \quad r_n^2 = n\lambda R$$

$$D_n^2 = 4n\lambda R$$

$$\boxed{D_n = 2\sqrt{n\lambda R}} \qquad (D_n \propto \sqrt{n}).$$

$\therefore$ Diameters of dark fringes are in ratio of square root of natural number.

$$D_1 : D_2 : D_3 : \dots = \sqrt{1} : \sqrt{2} : \sqrt{3} \dots$$

Q. The Newton's ring are observed normally in reflected light of wavelength 6000Å, the diameter of 10th dark ring is 0.50 cm, find the radius of curvature of length and the thickness of the fringe.

Diameter

∴ For dark fringe :

$$D_n = 2\sqrt{n\lambda R}$$

$$\Rightarrow 0.50\times10^{-2} = 2\sqrt{6000\times10^{-10}\times10\times R}$$

$$\therefore R = 1.06 \text{ cm.}$$

For dark fringe

$$2t = n\lambda$$

$$\Rightarrow D_n = 2\sqrt{2tR}$$

$$\therefore 0.50\times10^{-2} = 2\sqrt{2\times t\times1.06\times10^{-2}}$$

$$t = 3\times10^{-4} \text{ cm}$$ ans.

Q. The lower surface of a lens resting on a plane glass, a fringe of radius of curvature of 400 cm when illuminated monochromatic light the arrangement produce Newton's ring. The 15th bright rings has a diameter of 1.16 cm. Calculate the wavelength of the monochromatic light.

$R = 400$ cm
$n = 15$
$D_n = 1.16$
$\lambda = ?$

∴ Diameter for bright fringe :

$$D_n^2 = (2n-1)\times2\lambda R$$

$$\Rightarrow R = \frac{D_n^2}{2(2(n)-1)R} = \frac{(1.16\times10^{-2})^2}{2(2(10)-1)} = 8.85\times10^{-7}\text{ m}$$

Ans.

Q. In Newton's Ring experiment the diameter 4th and th dark fringes are 0.400 cm and 0.700 cm respectively. Find the diameter of 20th dark ring.

$$\therefore D_{n+p}^2 - D_n^2 = 4(n+p)\lambda R - 4n\lambda R = 4p\lambda R$$

$$\Rightarrow D_{12}^2 - D_4^2 = 4\times(8)\times\lambda\times R$$

$$(0.7)^2 - (0.4)^2 = 4\times8\times\lambda\times R$$

$$\frac{(0.3)(1.1)}{4\times8} = \lambda R$$

For

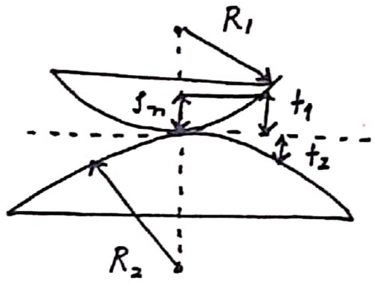
$$D_{20}^2 = 4\times20\times\lambda R$$

$$= \cancel{4}\times\overset{5}{\cancel{20}}\times\frac{0.33}{\cancel{4}\times\underset{2}{\cancel{8}}}$$

$$D_{20} = \sqrt{0.825} = 0.908 \text{ cm}$$ Ans

Newton's Ring Experiment with two curved surfaces.

Case 1: Plano convex on plano convex



From diagram;

$$t = t_1 + t_2$$

and $t = \dfrac{\int_n^2}{2R_1} + \dfrac{\int_n^2}{2R_2}$

$\therefore$ $2t = \dfrac{\int_n^2}{R}$ for a circle (Sagitta Theorem).

$$\Rightarrow \quad 2t = \frac{\int_n^2}{R_1} + \frac{\int_n^2}{R_2}$$

$$\int_n = \frac{D_n}{2}$$

$\Rightarrow$ For dark fringe $(2t = n\lambda)$

$$n\lambda = \frac{\int_n^2}{R_1} + \frac{\int_n^2}{R_2}$$

$$\therefore \quad n\lambda = \frac{D_n^2}{4}\left[\frac{1}{R_1} + \frac{1}{R_2}\right]$$

$$\frac{1}{R_1} + \frac{1}{R_2} = \frac{4n\lambda}{D_n^2}$$

$\longrightarrow$ Diameter of ring

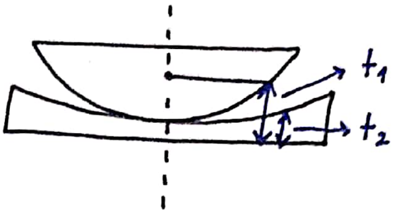
$\Rightarrow$ For bright fringes $\left(2t = (2n-1)\frac{\lambda}{2}\right)$

$$\therefore \quad (2n-1)\frac{\lambda}{2} = \frac{\int_n^2}{R_1} + \frac{\int_n^2}{R_2}$$

$$(2n-1)\frac{\lambda}{2} = \frac{D_n^2}{4}\left[\frac{1}{R_1} + \frac{1}{R_2}\right]$$

$$\Rightarrow \quad \frac{1}{R_1} + \frac{1}{R_2} = \frac{2(2n-1)\lambda}{D_n^2}.$$

Case 2 : Plano convex on plano concave



$\therefore \quad t = t_1 - t_2$

$\therefore \quad t = \frac{\rho_n^2}{R_1} - \frac{\rho_n^2}{R_2}$

$\rho_n = \frac{D_n}{2}$

$2t = \frac{\rho_n^2}{R_1} - \frac{\rho_n^2}{R_2}$

$\Rightarrow$ For dark fringe $(2t = n\lambda)$

$n\lambda = \rho_n^2 \left[\frac{1}{R_1} - \frac{1}{R_2}\right]$

$n\lambda = \frac{D_n^2}{4} \left[\frac{1}{R_1} - \frac{1}{R_2}\right]$

$\frac{1}{R_1} - \frac{1}{R_2} = \frac{4n\lambda}{D_n^2}$ ⟶ Diameter of ring

$\Rightarrow$ For bright fringes $\left(2t = (2n-1)\frac{\lambda}{2}\right)$

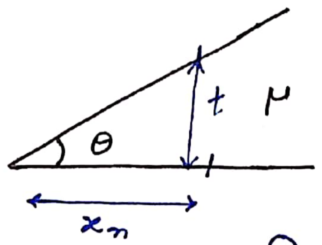
$(2n-1)\frac{\lambda}{2} = \rho_n^2 \left[\frac{1}{R_1} - \frac{1}{R_2}\right]$

$(2n-1)\frac{\lambda}{\cancel{2}} = \frac{D_n^2}{\cancel{4}\,2} \left[\frac{1}{R_1} - \frac{1}{R_2}\right]$

$\frac{1}{R_1} - \frac{1}{R_2} = \frac{2(2n-1)\lambda}{D_n^2}$

Interference of light in wedge shape thin film by normally reflected monochromatic light.

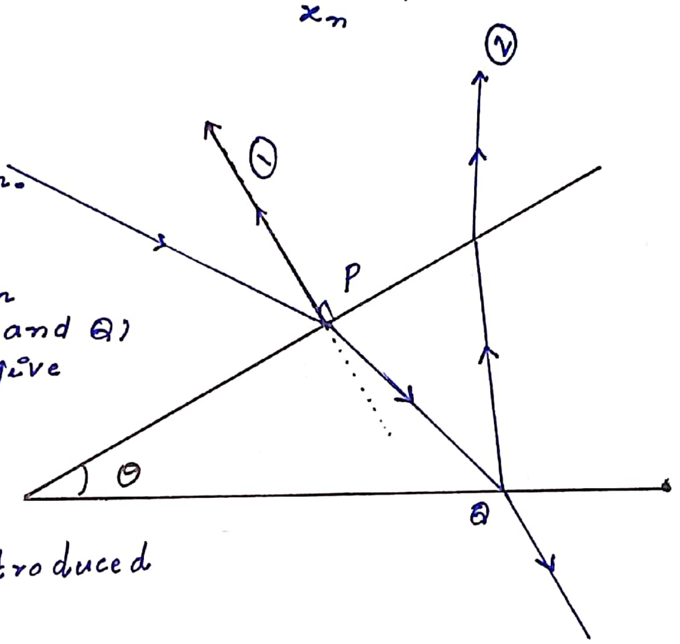
$x_n \rightarrow$ position of n^{th} fringe from wedge.



Here, $\mu \rightarrow$ refractive index of film.

$\theta \rightarrow$ Angle of inclination

$t \rightarrow$ thickness of film at x_n position.



When, light is reflected from upper and lower surface (P and Q) then they will interface to give fringes.

$\therefore$ The path difference introduced by film is '$2\mu t$'

Here $\frac{\lambda}{2}$ path difference is introduced in reflected light from upper surface

$$\therefore \quad \Delta = 2\mu t - \lambda/2$$

$\Rightarrow$ For maxima

$$\Delta = n\lambda$$

$$\therefore \quad n\lambda = 2\mu t - \frac{\lambda}{2}$$

$$\boxed{2\mu t = (2n+1)\frac{\lambda}{2}}$$

$$\boxed{t = \frac{(2n+1)\lambda}{4\mu}}$$

$\Rightarrow$ For minima

$$\Delta = (2n-1)\frac{\lambda}{2}$$

$$(2n-1)\frac{\lambda}{2} = 2\mu t - \frac{\lambda}{2}$$

$$\cancel{2}n\frac{\lambda}{2} = \cancel{2}\mu t$$

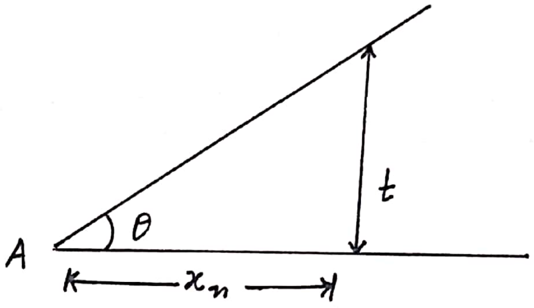
$$\boxed{2\mu t = \frac{n\lambda}{2}}$$

$$\boxed{t = \frac{n\lambda}{4\mu}}$$

$\Rightarrow$ Fringe width

→ For dark fringe:

$$2\mu t = n\lambda$$



$$\frac{t}{x_n} = \tan\theta$$

$\therefore$ $$2\mu x_n \tan\theta = n\lambda$$

for small angles

$$\tan\theta \approx \theta = \frac{\text{arc}}{\text{radius}}$$

$$2\mu x_n \theta = n\lambda$$

For $(n+1)^{th}$ fringe:

$$2\mu (x_{n+1})\theta = (n+1)\lambda$$

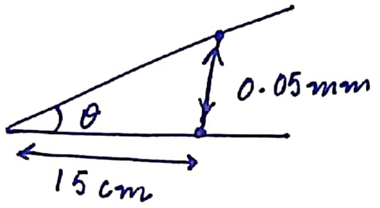
$\Rightarrow$ $$2\mu (x_{n+1} - x_n)\theta = \lambda$$

and $\beta = x_{n+1} - x_n$

$\Rightarrow$ $$\boxed{\beta = \frac{\lambda}{2\mu\theta}}$$

Q. Two glass encloses a wedge shaped air film touching at the edge and are seperated by 0.05 mm diameter at a distance of 15 cm from edge. Calculate the fringe width of monochromatic light 6000Å from a broad pass normally on the field.

$\therefore \quad \theta = \tan\theta = \dfrac{0.05\times10^{-3}}{15\times10^{-2}}$



Hence;

$$\beta = \frac{\lambda}{2\mu\theta} \qquad \mu = 1 \text{ for air}$$

$$\beta = \frac{6000\times10^{-10}}{2\times1\times\dfrac{0.05\times10^{-3}}{15\times10^{-2}}} = 9\times10^{-4}\,m = 0.09\,cm \quad \text{ans.}$$

Q. Light of wavelength 6000Å falls normally on thin wedge shaped film of refractive index 1.4 forming fringes that are 2.0 mm apart find the angle of wedge in second.

$\therefore \quad \lambda = 6000\times10^{-10}\,m$

$\mu = 1.4$

$\beta = 2\times10^{-3}\,m$

$\therefore \quad \theta = ?$

$1^\circ = 60'$

$1' = 60''$

$\therefore \quad 1^\circ = 60\times60''$

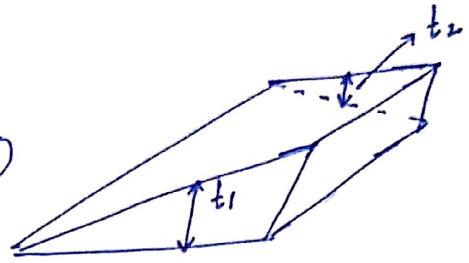
$$\Rightarrow \quad \beta = \frac{\lambda}{2\mu\theta}$$

$$\theta = \frac{\lambda}{2\mu\beta} = \frac{6000\times10^{-10}}{2\times1.4\times2\times10^{-3}} = 1.07\times10^{-4}\,rad$$

$$\therefore \quad \theta = \left(1.07\times10^{-4}\times\frac{180}{\pi}\times60\times60\right) \text{second}$$

$$\theta = 22.07 \text{ seconds} \quad \text{Ans.}$$

Q. A square piece hollow film with $\mu = 1.5$ has a wedge section so that if the thickness of two opposite side is t_1 and t_2. If the no. of fringes appearing with wavelength $\lambda = 6000\text{Å}$ is 10. Calculate $t_1 - t_2$.



$\therefore \quad \Delta = 2\mu t - \lambda/2$

$\therefore \quad n\lambda = 2\mu t_1 - \lambda/2 \quad — (1)$

$(n+10)\lambda = 2\mu t_2 - \lambda/2 \quad — (2)$

Subtracting (1) and (2)

We get

$2\mu\,(t_1 - t_2) = 10\lambda$

$t_1 - t_2 = \dfrac{\cancel{10}^{\,5} \times \cancel{6000}^{\,20000} \times 10^{-8}}{\cancel{2} \times \cancel{1.5}\ \cancel{0.5}}$

$t_1 - t_2 = 2000 \times 10^{-7}$ cm Ans.

These handwritten notes are of PHY-S102 taught to us by Prof. Prabal Pratap Singh, compiled and organized chapter-wise to help our juniors. We hope they make your prep a bit easier.

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